AN EXPERIMENTAL INVESTIGATION OF TELLURIC AND MAGNETO-TELLURIC PHENOMENA

by

THOMAS FERRIS WEBSTER
EDMONTON, Alberta.
September 1957.

FACULTY OF ARTS AND SCIENCE

DEPARTMENT OF PHYSICS

UNIVERSITY OF ALBERTA

ABSTRACT

The relation between natural electrical currents flowing in the earth, or telluric currents, and subsurface structure was studied using procedures developed by previous investigators. The scale of the study was larger than had been attempted previously, and the results were not as conclusive. It was found that the telluric currents are limited in their application to subsurface exploration by the non-uniformity of the telluric field over very large areas. This practical limit of telluric exploration is thought to be several hundred miles.

A second method of subsurface exploration was studied using electromagnetic induction relationships between telluric currents and the earth's magnetic field at the surface of the earth. The relation between electromagnetic induction and subsurface structure had been predicted theoretically but no experimental results have been published. It was found that measurements of the necessary order of accuracy were possible, but that the method cannot be thoroughly tested until new procedures are developed for analysing the data.

THE UNIVERSITY OF ALBERTA

AN EXPERIMENTAL INVESTIGATION OF TELLURIC AND MAGNETO-TELLURIC PHENOMENA

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR

THE DEGREE OF MASTER OF SCIENCE

Faculty of Arts and Science

Department of Physics

by

Thomas Ferris Webster

Edmonton, Alberta

September, 1957

TABLE OF CONTENTS

CHAPTER		PAGE
I	INTRODUCTION	1
II	THEORY	
	a) The Earth's Magnetic Field	5
	b) Telluric Current Theory	9
	c) Magneto-telluric Theory	16
III	FIELD METHODS	
	a) Telluric Prospecting	26
	b) Magneto-telluric Prospecting	35
IV	METHODS OF ANALYSIS	
	a) Analysis of Telluric Records	44
	b) Analysis of Magneto-telluric Records	50
V	ANALYSIS AND INTERPRETATION	
	a) Analysis of a Typical Telluric Record	59
	b) Interpretation of the Telluric Analysis	65
	c) Analysis of a Typical Magneto-telluric Record	74
	d) Interpretation of the Magneto-telluric Analysis	81

CHAPTER		PAGE
VI	CONCLUSIONS	
	a) Summary of Findings	87
	b) Recommendations	90
	BIBLIOGRAPHY	93

LIST OF TABLES

TABLE		PAGE
I	Magnetic Field Corresponding to a Telluric Field of One Millivolt per Kilometer	36
II	Analysis of Telluric Records, Clairmont, Alberta	63
III	Results of Telluric Analysis	66

LIST OF FIGURES

FIGURE		PAGE
1.	Depth of Electromagnetic Penetration as a Function of Period	19
2.	Magnetic Intensity Corresponding to an Electric Field of One Millivolt per Kilometer	21
3.	Location of Telluric and Magneto-telluric Field Stations Across the Canadian Prairies	27
4.	Telluric Record With Input Impedance too High	29
5.	Typical Electrode Installation	32
6.	Telluric Recording Apparatus During Operation	34
7.	Recording Magnetometer Circuit	38
8.	Detecting Unit, Flux-gate Magnetometer	40
9.	Typical telluric Record Obtained at Field Station	45
10.	Typical Telluric Record Obtained at Base Station	46
11.	Apparent Resistivity for Two-layered Subsurface	56

FIGURE		PAGE
12.	Phase Difference Between Components for Two-layered Subsurface	57
13.	Field Station Ellipse, Clairmont	64
14.	Ellipse Areas Across Precambrian Profile	67
15.	Relative Telluric Amplitudes Across Precambrian Profile	68
16.	Telluric Record, Meadow Lake	75
17.	Magnetic Record, Meadow Lake	76
18.	Average Telluric and Magnetic Cycles, 0050-0101, June 5/57	77
19.	Apparent Resistivity, Molanosa	82
20.	Apparent Resistivity, Meadow Lake	83
21.	Apparent Resistivity, Vega	84

ACKNOWLEDGMENTS

I wish to thank the following:

Dr. G. D. Garland, for suggesting this project to me, and for his advice and guidance during its completion.

Dr. J. H. Harrold for assistance in producing the diagrams in the thesis.

The California Standard Company, of Calgary, for the California Standard Graduate Fellowship.

CHAPTER ONE

INTRODUCTION

The natural electromagnetic field of the earth is affected at the surface by the electrical properties underground. It is the purpose of this research to investigate experimentally two particular methods of subsurface exploration using the earth's electromagnetic field.

The investigations of each of the two methods of exploration are developed in a parallel manner throughout the thesis. The first method investigated is that of Telluric Prospecting. This method uses the variations in the electric currents flowing in the earth -- "telluric currents" -- to determine subsurface structure. The second method is called "magneto-telluric prospecting" and uses the relation between the electric and magnetic components of the electromagnetic field at the surface for determining subsurface structure.

Telluric prospecting has been attempted in Europe[1,2] and the United States.[3,4] The scale of

---

[1] Schlumberger, Trans. Am. Geoph. Union, 3:271 (1939)

the prospecting was comparatively small -- less than one hundred miles across the area covered. The results obtained showed a correlation between the telluric currents at the surface and subsurface structure. The present application of the method of telluric prospecting was applied over a larger scale (about one thousand miles) and was an attempt to correlate telluric currents across the Canadian Prairies with the depth of the Precambrian basement.

Very few details have been published in English about methods of telluric prospecting. Those which are available are concerned with local structural features. It is the purpose of this study to give details about techniques in telluric prospecting and to apply these techniques to a large scale structural feature. It was further hoped that a large scale study might yield more information about the nature of telluric currents in general over a large area.

---

[2] Schlumberger and Kunetz, C. R. Acad. Sci. Paris, 223:551-53 (1946).

[3] Dahlberg, Geophysics, 10:494 (1945).

[4] Boissannas and Leonardon, Geophysics, 13:387 (1948)

No results of magneto-telluric prospecting have been published up to the present. Therefore, it seems desirable that an experimental investigation of magneto-telluric prospecting be attempted following the outline of the theory developed. The theory has been developed by Cagniard,[5] who suggested that subsurface structure could be determined from phase and amplitude relationships between the components of the telluric and magnetic fields at the surface.

The depth of the Precambrian basement was used as a known geological feature in the magneto-telluric prospecting as in the telluric prospecting. Experimental stations were established at locations which gave a wide range of depth of the Precambrian basement. The results of the magneto-telluric exploration were compared with the known geologic structure to determine the success of the method as applied.

In the remainder of the thesis, the corresponding features of each method are outlined in a parallel manner. The following three chapters describe the theory of the electromagnetic field, the details of the field methods used, and the

---

[5] Cagniard, Geophysics, 18:605, (1953)

methods used in analyzing the data obtained. Typical cases of each method of prospecting are analyzed in Chapter Five, and the results of the overall explorations are interpreted.

CHAPTER TWO

THEORY

a) The Earth's Electromagnetic Field

The magnetic field of the earth is constantly varying. It can be broken up into two portions: the ambient, or dipole magnetic field, which varies only slowly over the course of many years, and the varying magnetic field, composed of both irregular random variations and a number of periodic regular variations.

The "earth's electromagnetic field" as used here refers to the varying magnetic field plus the varying electric currents which flow in the earth. The electric and magnetic fields, considered together, make up the earth's electromagnetic field.

The connection between the telluric currents and the magnetic variations becomes evident when the source of the electromagnetic field is considered. The location of the source of such fields may be determined by means of a spherical harmonic analysis.[6] Such an analysis was used by Gauss to

---

[6] Chapman and Bartels, Geomagnetism, p.606, Oxford, 1940.

show that the dipole magnetic field is of internal origin. A similar analysis applied to the varying magnetic field shows that it is about two-thirds due to sources which are located outside the earth itself, and about one-third due to sources inside the earth.[7] The internal portion of the magnetic variation can reasonably be explained as due to electric currents induced within the earth by the external portion of the magnetic variation. The external sources of the magnetic variation are believed to be vast current sheets located in the ionosphere of the earth.

The source of the electromagnetic field, therefore, is situated at a great distance away from the earth. Cagniard[8] points out that the length of the wave of a disturbance is enormous. For a disturbance of thirty second period, the wave length would be nine million kilometers, which is fourteen hundred times the radius of the earth. Such a wave would extend over the whole earth. This widespread nature of the electromagnetic disturbances ensures a

---

[7] Ibid., p. 684

[8] Cagniard, Handbuch Der Physik, p. 462, Berlin, 1956

uniformity over a large area of the earth.

The varying electromagnetic field may be observed on the surface of the earth. It is seen as a continually varying phenomenon whose variations have a wide frequency range, and which persist for varying lengths of time. The direction of the varying field is continually changing. Because it is expensive and complicated to measure all of the independent components of the field simultaneously, it is usual to measure only a few of the components. Therefore, only the horizontal components of the electric and magnetic fields at the surface will be considered in the following.

Two types of prospecting using the earth's electromagnetic field will be considered in detail. The first, known as telluric prospecting, uses point-to-point variations in the telluric currents to give subsurface information. The second type, known as magneto-telluric prospecting, measures the magnetic and telluric components simultaneously at one station. It is thought that subsurface information can be obtained from amplitude and phase relationships between the electric and magnetic components. The theory and application of each of these methods

will be treated in turn.

b) Telluric Current Theory

It is found that a varying voltage exists between two electrodes inserted in the ground. This varying voltage is associated with sheets of electric currents in the earth, known as telluric currents. It is usual to refer to the measurement of these voltages as measurements of the telluric current. This practice will be followed.

Telluric currents, when simultaneously measured over a part of the earth's surface, show similarities at different locations. However, though similar, earth currents do show variations with location which are thought to be due to subsurface electrical properties. If this is true, then simultaneous earth current measurements over an area should reveal indications of subsurface structure.

To measure earth currents, two conducting electrodes are placed in the ground at distances apart which may vary from a few hundred feet to several miles. A voltage recording device is placed between the electrodes. This gives a record of the potential difference between the two electrodes, which may be interpreted as a record of

voltage gradient in the direction of the line joining the two electrodes. Voltage gradient is a vector quantity with both magnitude and direction. To determine the potential gradient vector, it is necessary to measure two components, preferably at right angles.

Telluric currents flow in large sheets. These sheets are uniform over a large area at a given time, but are constantly varying with time, both in amplitude and direction. Because it would be far too expensive to measure the telluric currents simultaneously at a great number of stations, it is usual to measure the telluric currents only at a fixed base station and at a field station which can be mobile. If all the field station records are compared with those at the base station, results can be obtained as if the telluric currents were measured simultaneously at a large network of stations. The electrical properties of the subsurface can then be determined for the area covered by the survey.

Assuming that the earth current sheet is continuous, there will be a linear relation between the telluric vectors at the base station and the

field station.[9] Letting the components of the telluric field at the base station be x and y, and at the field station X and Y, then the linear relation may be written:

$$\begin{aligned} X &= ax + by \\ Y &= cx + dy \end{aligned} \qquad (1)$$

Equations (1) are a linear vector transformation. The determinant, $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, is invariant under the selection of co-ordinate systems. That is, whatever the orientation of the axes used to measure the telluric currents at a station, the determinant A will be the same. Thus, A is associated with a particular location and is invariant.

The quantities expressed in equation (1), (viz., X, Y, x, and y) represent potential gradients measured between the electrodes. Measuring the value of the telluric field accurately is difficult, however, due to potentials developed at the electrodes. These "contact potentials" are super-imposed upon the natural telluric potentials. Since it is difficult to measure the value of the contact potentials, it is not practical to try to measure

---

[9] Porstendorfer, <u>Tellurik</u>, p.51, Berlin, 1954.

the total telluric field accurately.

Instead, measurements are confined to variations in the telluric field. The linear relationships of equation (1) will be true for changes in the field.[10] Thus, we may write, expressing the differences in the field quantities as increments:

$$\begin{aligned} \triangle X &= a \triangle x + b \triangle y \\ \triangle Y &= c \triangle x + d \triangle y \end{aligned} \qquad (2)$$

Equations (2) are similar to equations (1) and as before, A, the determinant of the coefficients, is unique at a given location. The elements of the determinant are obtained by solving equations (2), using as values of the increments, changes in the field quantities measured during some arbitrary time interval.

Each set of corresponding base and field station readings must be standardized so that the data from various stations may be compared. There will be two sets of readings corresponding to the quantities in equation (2): the readings at the base station and the readings at the field station. To standardize the two sets of readings, it is most

---

[10] Ibid., p. 52.

convenient to normalize the base vectors so that when represented vectorially, the end points lie on a unit circle. To do this, we set

$$\Delta z^2 = \Delta x^2 + \Delta y^2 \qquad (3)$$

Then let

$$X' = \frac{\Delta X}{\Delta z}$$

$$Y' = \frac{\Delta Y}{\Delta z}$$

$$x' = \frac{\Delta x}{\Delta z}$$

$$y' = \frac{\Delta y}{\Delta z}$$

In terms of these new quantities, equation (2) becomes

$$\begin{aligned} X' &= ax' + by' \\ Y' &= cx' + dy' \end{aligned} \qquad (4)$$

and

$$x'^2 + y'^2 = 1 \qquad (5)$$

Equation (5) indicates that the base station electric vector lies upon a unit circle. Under the same operations, the field station vector will have a characteristic form. This can be found by solving equation (4) for x' and y'.

$$x' = \frac{dX' - bY'}{ad - bc}$$

$$y' = \frac{aY' - cX'}{ad - bc} \qquad (6)$$

Combining equations (5) and (6) gives

$$(dX' - bY')^2 + (aY' - cX')^2 = ad - bc$$

This is the equation of an ellipse. The normal vector at the field station thus describes an ellipse when the normal vector at the base station describes a unit circle. The area of the ellipse is $S = \pi(ad - bc) = \pi A$, where A is the determinant of the linear vector transformation describing the relation between the telluric field at the field station and at the base station. A is thus the ratio of the area of the ellipse at the field station to the area of the unit circle at the base station. Since the determinant A is independent of the choice of axes, the ellipse area is independent of the axes used to measure the electric field. Therefore, the area and orientation of the field ellipse at any point is independent of the direction of the electric field.

The form of the field vector ellipse gives information about the anisotropic nature of the subsurface electrical resistance. In accordance

with Ohm's law, the greatest potential gradient on the earth's surface is in the direction in which the current encounters the greatest resistance. This direction is that of the major axis of the ellipse. Perpendicular to this, the current encounters the least resistance, and this is the direction of the minor axis of the ellipse. An especially high apparent resistance would produce a large ellipse area. Thus, the absolute magnitude of the electric vector may be interpreted as a relative quantity expressing the apparent resistance of the soil.

The special structural case of this research is that of two layers, a sedimentary layer overlying a highly resistant crystalline basement. With this type of structure, the area of the ellipse is inversely proportional to the thickness of the sedimentary layer. Thus, a profile or a map showing variations in the area of the ellipses over some area should give a qualitative picture of the upper surface of the basement below that area.

c) Magneto-telluric Theory

The general purpose of the magneto-telluric prospecting which was investigated is to obtain structural information by matching measured values of the magnetic and telluric fields at the surface with theoretical values. Before this can be done, the theoretical relationships between the components of the electromagnetic field at the surface must be developed. To do this, certain properties of both the electromagnetic field and the subsurface structure are assumed and under these assumptions the theoretical relations are obtained.

First, it is assumed that the variations in the magnetic field induce the telluric currents. It is further assumed that the induced current sheet is uniform and in one direction only. The assumption of uniformity is necessary and is verified by experiment.[11] The assumption that the currents flow in one direction is made for convenience in development of the theory and does not result in any loss in generality.

With these assumptions, the currents at the

---

[11] Schlumberger and Kunetz, <u>Trans Am Geoph Union</u>, 3:271 (1939).

surface may be expressed as:[12]

$$I_x = \cos \omega t$$

$$I_y = I_z = 0$$

Then at depth z:

$$I_x = \exp(-z\sqrt{2\pi\sigma\omega})\ \cos(\omega t - z\sqrt{2\pi\sigma\omega})$$

$$I_y = I_z = 0$$

σ is the conductivity of the soil.

If p (depth of penetration) is the depth at which the amplitude is reduced to $\frac{1}{e}$ of its value at the surface, then

$$p = \frac{1}{\sqrt{2\pi\sigma\omega}}$$

It can be seen that as ω, the angular frequency increases, the depth of penetration becomes less. That is, phenomena of a short period have less effect at depth than those of a longer period. This characteristic of electromagnetic quantities is known as the "skin effect." It has a marked effect on magneto-telluric relationships, since the subsurface effects of an electromagnetic phenomenon decrease inversely as the period of the

---

[12] The theoretical development follows the outline of Cagniard, Geophysics, 18:605 (1953)

phenomenon. The depth of electromagnetic penetration as a function of period is shown in Figure 1.

The relation between the electric and magnetic fields at the surface of the earth may be developed using the Hertzian vector, Π, which satisfies Maxwell's equations. In terms of the Hertzian vector, Maxwell's equations give for a horizontally stratified earth:

$$\nabla^2 \Pi + 4\pi\sigma\omega i \Pi_x = 0 \qquad (9)$$

$$H = 4\pi\sigma \operatorname{curl} \Pi$$

or $$H_y = 4\pi\sigma \frac{\partial \Pi}{\partial z}$$

$$H_x = H_z = 0 \qquad (10)$$

$$E = \operatorname{grad} \operatorname{div}\Pi - \nabla^2 \Pi$$

so that $$E_x = 4\pi\sigma\omega i \Pi_x$$

$$E_y = E_z = 0 \qquad (11)$$

Since E_x is proportional to Π_x, E_x may be chosen as the Hertzian vector without loss of generality. Then (9) is:

$$\frac{\partial^2 E_x}{\partial z^2} + 4\pi\sigma\omega i E_x = 0$$

and $$H_y = -\frac{i}{\omega}\frac{\partial E_x}{\partial z}$$

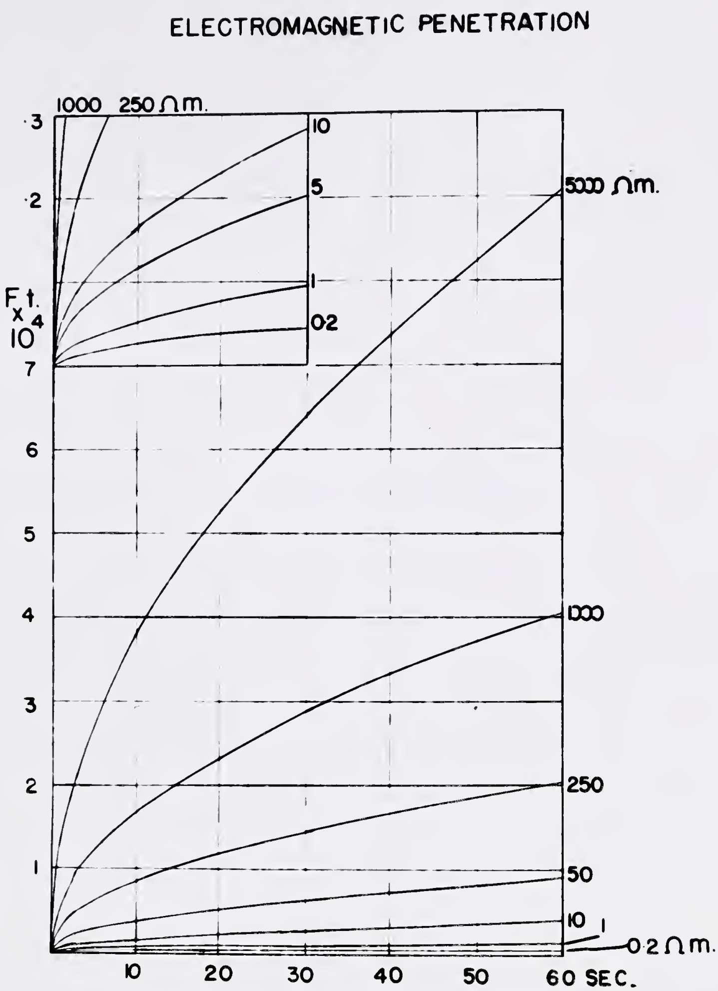

FIGURE 1

DEPTH OF ELECTROMAGNETIC PENETRATION AS A FUNCTION OF PERIOD (After Cagniard)

E_x has the form

$$E_x = A \exp(a\sqrt{\sigma} z) + B \exp(-a\sqrt{\sigma} z)$$

where $$a = 2\pi\sqrt{\frac{2}{T}} \exp(-i \frac{\pi}{4})$$

If the subsurface is homogeneous, the relation between the electric and magnetic vectors will be:

$$\frac{E_x}{H_y} = \sqrt{\frac{\rho}{2T}} \qquad (12)$$

In equation (12), the value of the resistivity, ρ, is the true resistivity of the subsurface if the subsurface is homogeneous. For the more general case of a stratified subsurface, however, the resistivity of the soil will increase with depth so that there will not be a unique resistivity. The value of ρ obtained from equation (12) is known as the "apparent resistivity."

Equation (12) is shown graphically in Figure 2. In this figure, the intensity in gammas (one gamma equals 10^{-5} oersteds) corresponding to an electric field of one millivolt per kilometer is shown for various resistivities.

In the general case, the subsurface will be horizontally stratified with layers whose resistivity

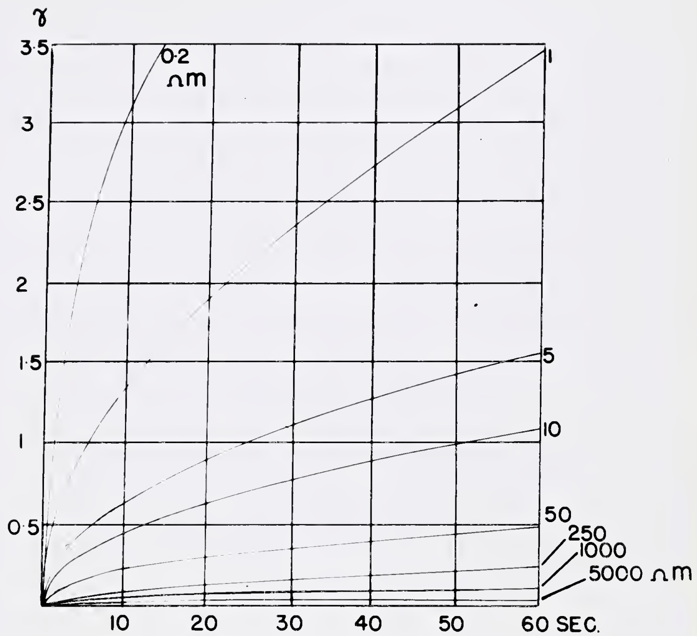

FIGURE 2

MAGNETIC INTENSITY CORRESPONDING TO AN ELECTRIC FIELD OF ONE MILLIVOLT PER KILOMETER (After Cagniard)

will increase with depth. Because of the skin effect, the longer period variations will be more affected by the higher resistivity of the deeper layers than will the shorter period variations. Thus the apparent resistivity computed from variations with long periods will be greater than for those with short periods. That is, the apparent resistivity is a function increasing with period if the underground is horizontally stratified.

The particular case to be dealt with here is that of two layers: a sedimentary layer of low resistivity above a highly resistive crystalline layer. This roughly corresponds to the condition across the Canadian Prairies, where several thousand feet of sedimentary layers overlie the crystalline Precambrian basement. While it is true that the sedimentary layers do not have homogeneous electrical properties, the resistivity of the Precambrian basement is greater than the resistivity of the overlying sediments by a factor of the order of 10^6. The greatest variation of resistivity in the sediments, on the other hand, is usually less than a factor of 10^2 between the upper and lower sedimentary layers.

The electromagnetic relations at the surface corresponding to the two-layer case may be determined by using the values of (13) and (14) and solving for two layers. In the upper layer:

$$E_x = A \exp(a\sqrt{\sigma_1} z) + B \exp(-a\sqrt{\sigma_1} z)$$

$$H_y = \exp(i\tfrac{\pi}{4}\sqrt{2\sigma_1 T})(-A \exp(a\sqrt{\sigma_1} z) + B \exp(-a\sqrt{\sigma_1} z) \tag{15}$$

In the lower layer:

$$\begin{aligned} E_x &= \exp(-a\sqrt{\sigma_2} z) \\ H_y &= \exp(i\tfrac{\pi}{4}\sqrt{2\sigma_2 T}) \exp(-a\sqrt{\sigma_2} z) \end{aligned} \tag{16}$$

The subscripts 1 and 2 signify the upper and lower layers respectively. The conditions of continuity across the boundary enable the factors A and B to be determined:

$$A = \frac{\sqrt{\sigma_1} - \sqrt{\sigma_2}}{2\sqrt{\sigma_1}} \exp(-ah(\sqrt{\sigma_1} + \sqrt{\sigma_2}))$$

$$B = \frac{\sqrt{\sigma_1} + \sqrt{\sigma_2}}{2\sqrt{\sigma_1}} \exp ah(\sqrt{\sigma_1} - \sqrt{\sigma_2})$$

The fields at the surface can be expressed in terms of the depth of penetration, p, for layer 1 by:

$$E_x = M \exp - i\phi \tag{18}$$

$$H_y = \sqrt{2\sigma_1 T}\, N \exp i(\tfrac{\pi}{4} - \psi) \tag{19}$$

where

$$\begin{aligned} M\cos\phi &= \left(\frac{1}{p_1}\cosh\frac{h}{p_1}+\frac{1}{p_2}\sinh\frac{h}{p_1}\right)\cos\frac{h}{p_1} \\ M\sin\phi &= \left(\frac{1}{p_1}\sinh\frac{h}{p_1}+\frac{1}{p_2}\cosh\frac{h}{p_1}\right)\sin\frac{h}{p_1} \\ N\cos\psi &= \left(\frac{1}{p_1}\sinh\frac{h}{p_1}+\frac{1}{p_2}\cosh\frac{h}{p_1}\right)\cos\frac{h}{p_1} \\ N\sin\psi &= \left(\frac{1}{p_1}\cosh\frac{h}{p_1}+\frac{1}{p_2}\sinh\frac{h}{p_1}\right)\sin\frac{h}{p_1} \end{aligned} \tag{20}$$

The ratio of the electric component to the magnetic component is:

$$\frac{E_x}{H_y} = \frac{1}{\sqrt{2\sigma_1 T}}\,\frac{M}{N}\,\exp - i\left(\frac{\pi}{4}+\phi-\psi\right) \tag{21}$$

The phase difference between the telluric and the magnetic components is given by equation (21) and is $(\frac{\pi}{4}+\phi-\psi)$. The apparent resistivity, ρ_a, is:

$$\frac{1}{\sqrt{2\sigma_a T}} = \frac{M}{N}\,\frac{1}{\sqrt{2\sigma_1 T}}$$

or,

$$\rho_a = \rho_1\left(\frac{M}{N}\right)^2 \tag{22}$$

Equations (20) and (22) may be used to plot theoretical curves of apparent resistivity against time for given values of the resistivity of the two layers and the thickness of the upper layer. Equation (21) gives a similar theoretical curve for the phase difference between the telluric and magnetic components.

A possible method of magneto-telluric prospecting would be to measure two horizontal telluric and magnetic components at right angles. Using the values of these components, which would correspond to E_x and H_y, the apparent resistivity could be computed using equation (14). The values of apparent resistivity obtained could then be plotted against T, the period of the disturbance used in the calculations, and the result could be compared with the theoretical curves. The details of curve matching will be given in detail in the chapter on Methods of Analysis.

CHAPTER THREE

FIELD METHODS

a) Telluric Prospecting

In the summer of 1956, telluric measurements were made at a number of stations along a profile across the Canadian Prairies. (See Figure 3). The stations were chosen to be on the same geomagnetic latitude as the base station, the Dominion Observatory at Meanook, Alberta. At the westernmost station, Clairmont, Alberta, the top of the granitic Precambrian basement is 11,500 feet below the surface. This depth decreases towards the east as the thickness of the overlying sediments becomes less, and at Portage La Prairie , Manitoba, the top of the Precambrian is 2000 feet below the surface. The purpose of the telluric measurements was to attempt to relate the variations in the character of the telluric currents along the profile to the change in the thickness of the sedimentary layers.

The recorders used to measure the telluric field were two Varian type G-10. Recorders of this type work on a null-balance principle; that is, at balance they draw no current. The input voltage to

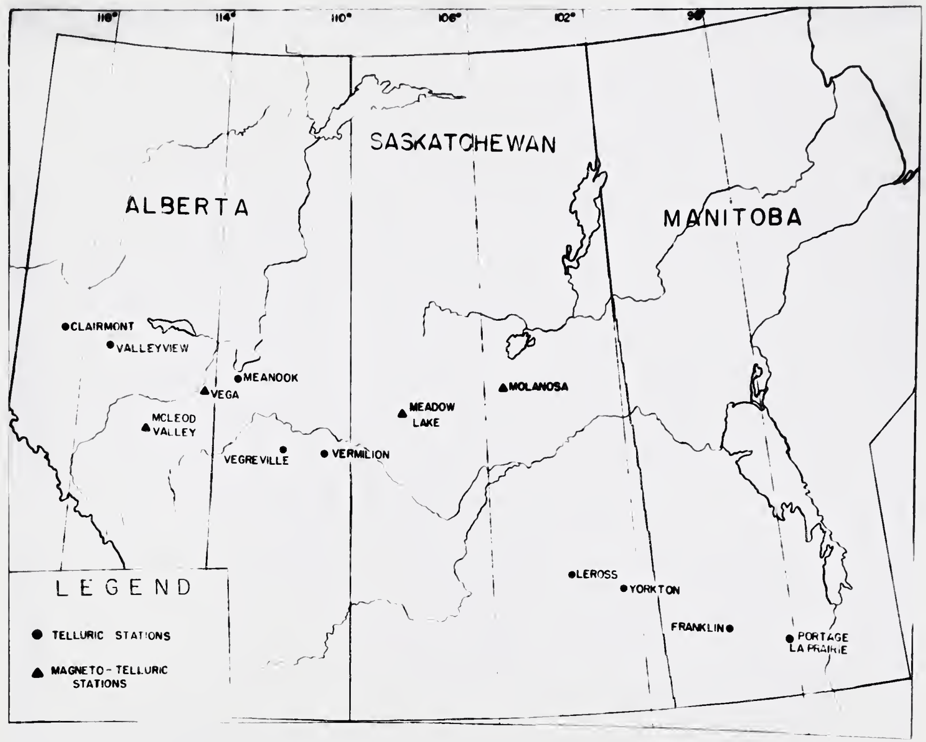

FIGURE 3

LOCATION OF TELLURIC AND MAGNETO-TELLURIC FIELD STATIONS

the recorder is compared with a standard cell voltage, and the difference voltage is amplified by a d-c amplifier. The d-c amplifier drives a servo-mechanism which moves a recording pen on a moving paper record. The response time of the recorders (the time required for the recording pen to travel the full width of the scale) was one second. In order to achieve critical damping of the system, the input resistance must be less than 0.1 megohm. Usually, the signal resistance between the electrodes is less than this value, but in some locations, notably those which were particularly dry, this resistance was exceeded, and usable records could not be obtained. In these instances, when the system cannot be critically damped, the recording pen will not follow the signal, and produces a record with square-shaped variations as shown in Figure 4.

The sensitivity of the recorders was stable enough to meet the requirements of field operations. The standard cells in the recorder, mercury hearing aid type cells, have a very low drop in E. M. F. throughout their life. They need to be replaced about every four months, and the sensitivity during

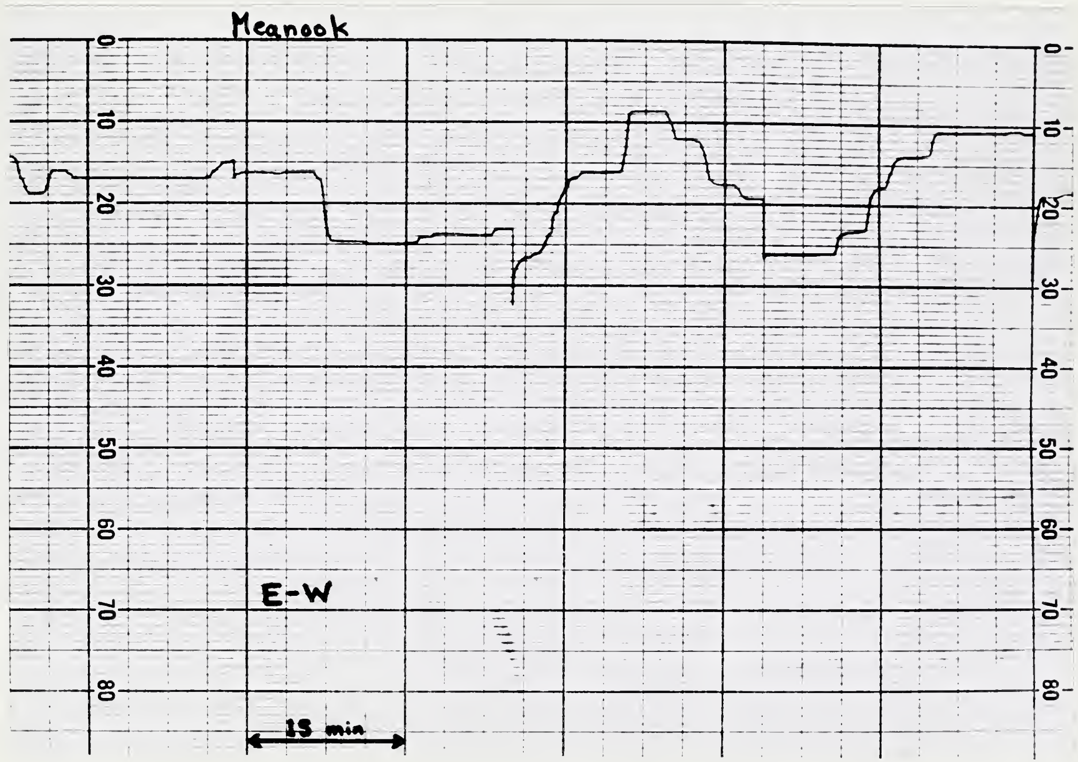

FIGURE 4

TELLURIC RECORD WITH INPUT IMPEDANCE TOO HIGH

this period is fairly stable.

The recorders had two available paper speeds: four inches per minute and four inches per hour. It was found that while the faster paper speed gave the high-frequency variations which the lower speed obscured, the difficulties in accurately timing the records made the fast paper speed impractical. Thus, all the records were obtained at a paper speed of four inches per hour. One of the recorders was used for the North-South component, and the other for the East-West. Timing marks were produced on the records by applying a small voltage to the recorders at regular intervals.

The records at the Meanook Observatory base station were taken on Brown recorders. These recorders have the advantage that the two telluric components are recorded on the same paper record, which reduces the error in timing.

The input terminals of the recorders were connected to the electrodes which were placed 1000 feet apart. The usual field practice was to install the four electrodes five hundred feet from the recording truck, which was at the centre of the cross formed by the cables going out to the elect-

rodes The cables used were five hundred feet lengths of number 18 rubber insulated, flexible copper wire. These cables were easily laid out and wound in at each location by the hand held reel shown in Figure 5.

The electrodes themselves were copper rods, immersed in an electrolyte of copper sulfate, which was contained in unglazed porous pots five inches deep. In order to maintain a uniform condition of the electrolyte, the solution was kept saturated. This type of electrode eliminates most of the contact potential, described in the previous chapter. In practice, it was found that there was a small, slightly varying contact potential as the electrolyte in the electrodes seeped out. This was possibly due partly to a change in the damping characteristics of the recorders as the resistance of the electrodes varied. Figure 5 shows an electrode installed in the ground.

The instruments were operated on 115 volts supplied by an ATR inverter operating from storage batteries. The advantage of the inverter-type supply is that it is cheap. However, it draws a heavy current from the storage batteries, which

FIGURE 5

TYPICAL ELECTRODE INSTALLATION

makes the voltage output drop with use. This variation in line voltage affected the paper speed and sensitivity of the recorders. Another disadvantage of the inverter-type supply is that it is not recommended for long period operation. Thus, the recordings with the inverter power supply were not continued for more than five hours.

Figure 6 shows the recording apparatus installed in the recording truck during a recording period. A voltmeter was used to make periodic checks on the line voltage, which was adjusted by a control on the inverter. The short-wave radio was used to receive time signals. The records were timed as accurately as possible to enable them to be compared with the corresponding records obtained at the Meanook Observatory.

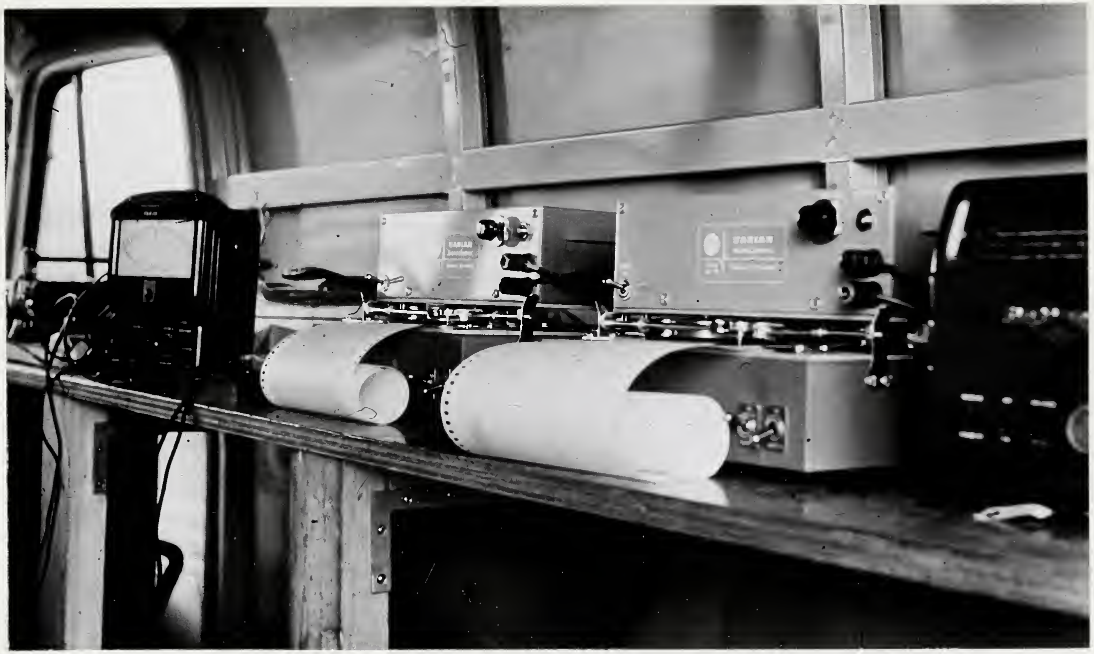

FIGURE 6

TELLURIC RECORDING APPARATUS DURING OPERATION

b) Magneto-telluric Prospecting

In the spring of 1957, magneto-telluric recordings were made at five locations in Alberta and Saskatchewan, shown in Figure 3. These stations were chosen to give a variation in the depth of the resistive basement below the surface. This depth of basement varied from 12,000 feet at McLeod Valley, Alberta, to 1,600 feet at Molanosa, Saskatchewan.

In the design of apparatus to measure the magneto-telluric components, it is necessary to have a magnetic detector which will record variations small enough to compare with the telluric variations. In Table I, the magnetic field in gammas (one gamma = 10^{-5} oersteds) corresponding to a telluric field of one millivolt per kilometer is given. Under typical conditions, the resistivity of the soil would be about fifty ohm-meters, and the telluric variation about ten millivolts per kilometer. From Table I, it can be seen that the magnetic variation corresponding will have an amplitude of eleven gammas for a variation with a period of five minutes. Thus, an adequate recording magnetometer should be able to give a sensitivity

TABLE I

(After Cagniard)

Magnetic Field in gammas corresponding to an electric field of one mV./ Km.

T = / ρ ↓	1 sec.	3 sec.	10 sec.	30 sec.	1 min.	2 min.	5 min.	10 min.	30 min.
0.2	1	1.73	3.16	5.48	7.75	11.0	17.3	24.5	42.4
1	0.447	0.775	1.41	2.45	3.46	4.90	7.75	11	19.0
5	0.2	0.346	0.632	1.10	1.55	2.19	3.46	4.90	8.49
10	0.141	0.245	0.447	0.775	1.10	1.55	2.45	3.46	6
50	0.0632	0.11	0.2	0.346	0.490	0.693	1.10	1.55	2.68
250	0.0283	0.049	0.0894	0.155	0.219	0.31	0.49	0.693	1.2
1000	0.0141	0.0245	0.0447	0.0775	0.110	0.155	0.245	0.346	0.6
5000	0.0063	0.111	0.02	0.0346	0.049	0.0693	0.11	0.155	0.268

which can be read at least to the nearest five gammas.

In order to measure variations of the order of at least five gammas, we must be able to measure variations which are only a tiny fraction of the total magnetic intensity. The horizontal component of the magnetic intensity at Meanook, for instance, is 13,000 gammas, so that five gammas is one part in twenty-five hundred. A small variation of this magnitude may be measured with a flux-gate magnetometer, which cancels out all but a small portion of the total magnetic field by means of a steady counter magnetic field.[13] The variations are then large enough relative to the remaining portion of the magnetic field to be measured.

The flux-gate magnetometer system consists of a detecting head, a phase sensitive detector and amplifier, a bucking circuit and a calibrating circuit. The phase sensitive detector circuit is shown in Figure 7. It is adapted from one described by Meek and Hector.[14]

---

[13] Wyckoff, Geophysics, 13:182 (1948).

[14] Meek and Hector, Can. Jour. of Physics, 33:364 (1955)

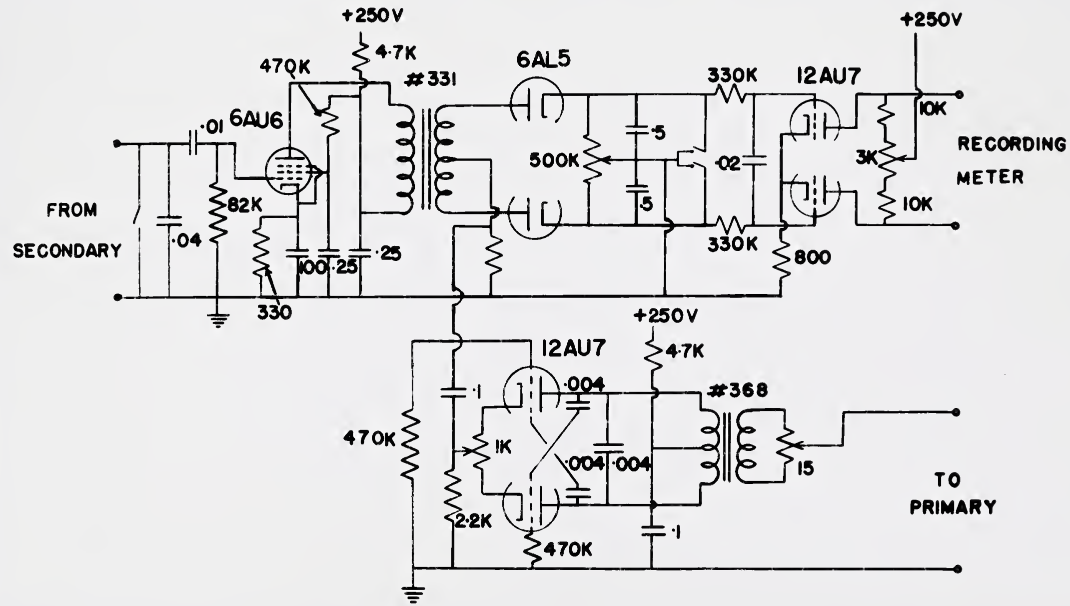

FIGURE 7

RECORDING MAGNETOMETER CIRCUIT

The detecting head of the magnetometer, shown in Figure 8, consists of two matched, mu-metal cores around which have been wound equal and oppositely-acting primary windings. A single secondary winding is wound outside of the primary coils, which are in series. The push-pull oscillator, shown in Figure 7, produces a signal of about five hundred cycles per second which is applied to the primary coils of the detecting head. When no magnetic field is present, no signal is detected by the secondary, as the primary coils are similar and oppositely wound. When an external field does exist, the primary applied magnetic field is shifted with respect to the hysteresis loop of the mu-metal cores. As the cores operate in opposite phases, the resultant magnetic flux produces a signal which is detected in the secondary at double the primary frequency. This signal is amplified and compared with double the oscillator frequency in the phase sensitive detector. The result is amplified in the d-c amplifier and used to operate a one milliampere recording meter.

The detecting head is mounted in the centre of a pair of Helmholtz coils, which are used to

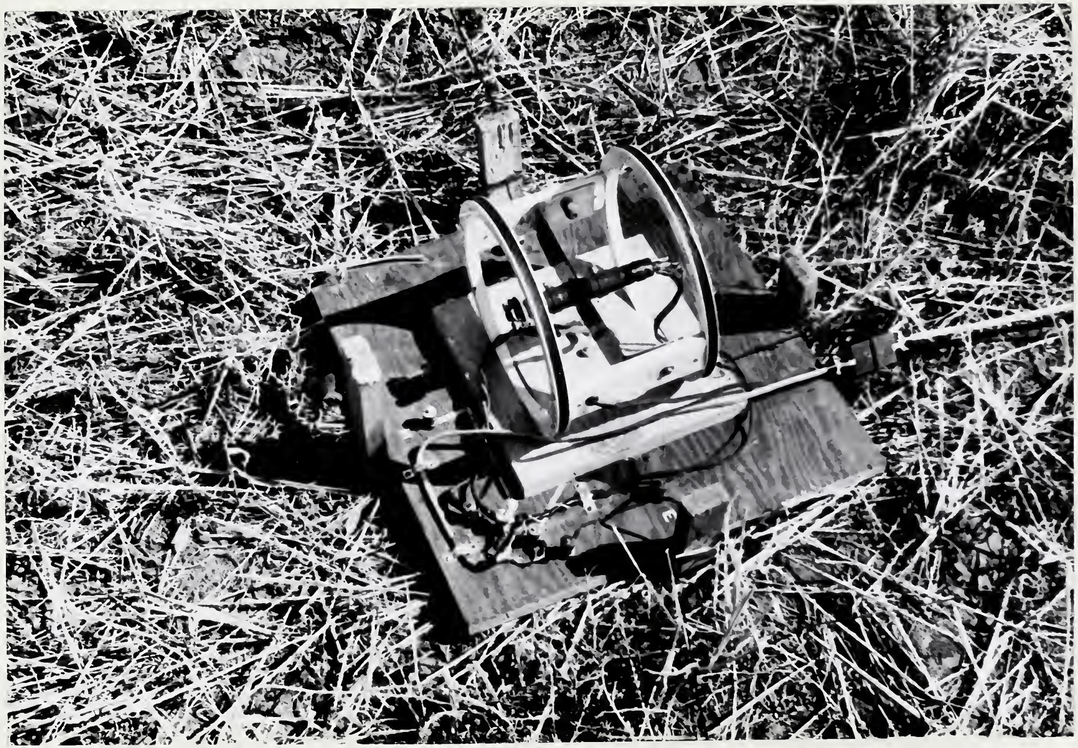

FIGURE 8

DETECTING UNIT, FLUX-GATE MAGNETOMETER

calibrate the magnetometer. The helmholtz coils can be seen in Figure 8. A small amount of current was applied to the calibrating helmholtz coils about every half hour during operation. Since the field in the region of the detecting head was known for a given current, the record could be calibrated. The calibrating mark also served as a time mark.

A further winding around the detecting head applied a bucking current which was adjusted so that only a small portion of the earth's field affected the magnetometer. Only a few milliamperes were necessary for this purpose. The bucking current was adjusted from time to time so that the recording magnetometer would not go off scale with large variations in the magnetic field. The bucking current was supplied by storage batteries and did not vary noticeably during the four or five days recording at each station.

The d-c output of the unit is designed to be used with a one milliampere recording meter. By using a voltage dividing system. the output could also be used with a Varian G-10 recording potentiometer as was used for the telluric measurements.

It was found that the Varian recorder gave a better record since the high frequency response of the unit reproduced high frequency disturbances well. The recording milliammeter, which draws a current, does not follow high frequency variations accurately. Most of the magnetometer records were obtained from the recording milliammeter, due to a failure of the Varian recorder.

The magneto-telluric recording apparatus was installed in a panel truck, and was powered by a fifteen hundred watt, two-cycle gasoline generator. The output from the generator was not steady enough to operate the recording apparatus, even though a regulating system was built into the output of the generator. In order to give more stability to the line voltage, the output of the generator was applied to a constant voltage transformer. In addition, the magnetometer circuit was provided with an electronic voltage control circuit. The high voltage for the magnetometer was supplied by a General Radio Type 1203A Unit Power Supply.

In operation, the detecting unit of the magnetometer was set up about two hundred feet away from the recording truck. Electrodes to measure

the telluric field were installed at right angles to the direction of the magnetometer and five hundred feet on each side. That is, the magnetometer detector was placed at the centre of the telluric cables and at right angles to the direction of the cables.

The component of the telluric field was recorded in the same manner as described for the telluric prospecting. However, the sensitivity of the Varian recorder was increased to ten millivolts full scale, which provided a more usable record than the previous sensitivity of fifty millivolts full scale. Since this higher sensitivity was too great to record on-scale during periods of intense electomagnetic activity, a two-hundred ohm attenuator was used to divide the signal input by a convenient factor. Using this attenuator, the source resistance could be standardized to two hundred ohms so that the recorder could always be critically damped.

CHAPTER FOUR

Methods Of Analysis

a) Analysis of Telluric Records

The area of the ellipse at a field station is obtained from a record such as the one shown in Figure 9. Only a single component is shown in this figure; the corresponding record of both components at the base station is shown in Figure 10. As was mentioned in the theory, changes in the telluric vector are used in the analyses of the records. The changes during any time interval are given a sign depending on whether the telluric current is increasing or decreasing during the interval. A positive difference, for instance, corresponds to an increasing current.

The first step in the analysis of the records is to read off the changes in the two components for a large number of time intervals. That is, for each time interval, Δt_i, the changes ΔX_i and ΔY_i will be obtained from the field records and the changes Δx_i and Δy_i from the base records. We have, from equation (3):

$$\Delta z_i^2 = \Delta x_i^2 + \Delta y_i^2$$

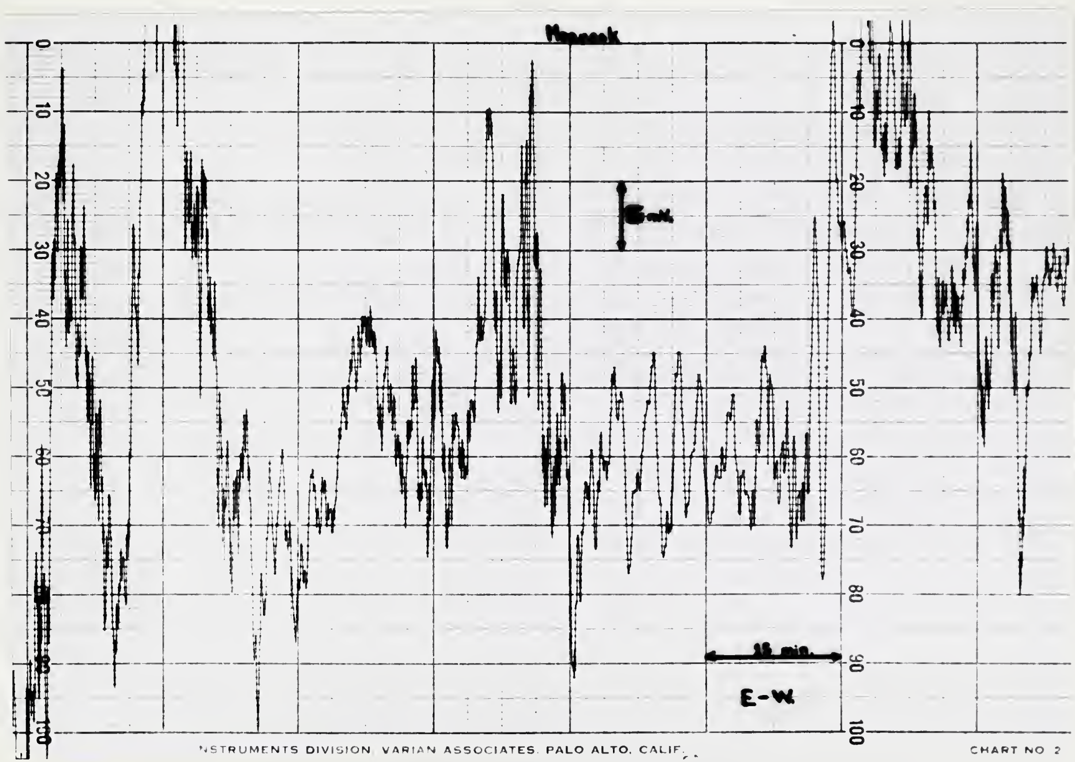

FIGURE 9

TYPICAL TELLURIC RECORD OBTAINED AT FIELD STATION

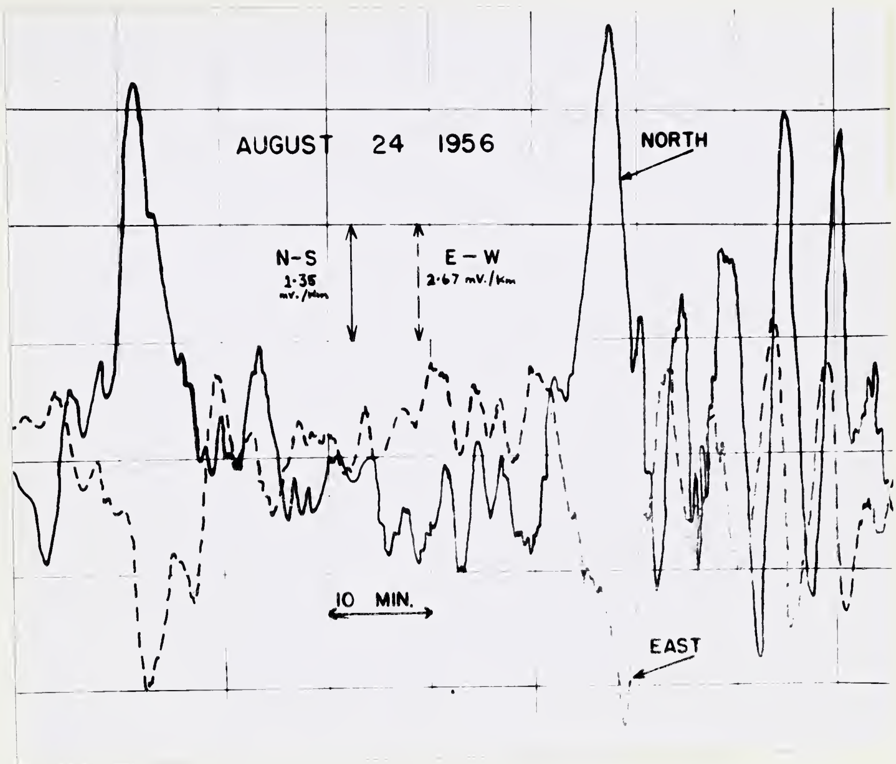

FIGURE 10

TYPICAL TELLURIC RECORD OBTAINED AT BASE STATION

and

$$x_i' = \frac{\triangle x_i}{\triangle z_i}$$

$$y_i' = \frac{\triangle x_i}{\triangle z_i}$$

$$X_i' = \frac{\triangle X_i}{\triangle z_i}$$

$$Y_i' = \frac{\triangle Y_i}{\triangle z_i}$$

Then, as was shown in the theory, the vectors x_i', and y_i' at the base station form the vector v_i' which lies on a unit circle. The vectors X_i' and Y_i' form the vector V_i' which lies on an ellipse at the field station.

In order to obtain a numerical method for the computation of the ellipse areas, let us consider the difference vectors for two different intervals, t_1 and t_2.[15]

Then, at the base station:

$$\triangle \vec{x}_1 + \triangle \vec{y}_1 = v_1$$

$$\triangle \overrightarrow{x2} + \triangle \overrightarrow{y_2} = v_2$$

And, at the field station:

---

[15] The method of analysis follows Porstendorfer, <u>Tellurik</u>, pp. 61-67, Berlin, 1954.

$$\Delta\vec{X}_1 + \Delta\vec{Y}_1 = \vec{V}_1$$

$$\Delta\vec{X}_2 + \Delta\vec{Y}_2 = \vec{V}_2$$

According to equation (1) then,

$$\Delta\vec{X}_1 = a\Delta\vec{x}_1 + b\Delta\vec{y}_1$$

$$\Delta\vec{Y}_1 = c\Delta\vec{x}_1 + d\Delta\vec{y}_1$$

$$\Delta\vec{X}_2 = a\Delta\vec{x}_2 + b\Delta\vec{y}_2$$

$$\Delta\vec{Y}_2 = c\Delta\vec{x}_2 + d\Delta\vec{y}_2$$

Forming the vector product of the two vectors:

$$|\vec{V}_1 \times \vec{V}_2| = \Delta X_1 \Delta Y_2 - \Delta X_2 \Delta Y_1$$

$$= (ad - bc)(\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$

Now, $S' = \frac{S}{\pi} = ad - bc$

$$= \frac{\Delta X_1 \Delta Y_2 - \Delta X_2 \Delta Y_1}{\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1} \qquad (21)$$

Therefore, in terms of the vector products of the base and field difference vectors, we can express the ratio of the ellipse area of the unit circle as:

$$S' = \frac{|\vec{V}_1 \times \vec{V}_2|}{|\vec{v}_1 \times \vec{v}_2|}$$

Using this equation, the area of the ellipse may be determined except for the factor π by forming the quotient of the vector product of the two difference vectors at the field and base stations.

The vectors $\vec{v_i}$ and $\vec{V_i}$ may be plotted to give a graphical picture of the relation between the unit circle and the field ellipse. It is difficult to determine the area of the ellipse by this method unless a large number of difference vectors are analyzed.

The computations involved in determining the ratio of the ellipse area to the unit circle are quite lengthy if the ratio is to be determined reliably. A simpler method of analysis is to compute the ratios of the absolute values of the corresponding difference vectors at the base and the field stations. That is, the ratios

$$\delta_i = \frac{|v_i|}{|V_i|} = \frac{\sqrt{\Delta x_i^2 + \Delta y_i^2}}{\sqrt{\Delta X_i^2 + \Delta Y_i^2}} \qquad (22)$$

may be determined for a large number of time intervals, and an average taken. The average value is a measure of the apparent specific resistivity of the soil.

b) Analysis of Magneto-telluric Records

The equations developed in the magneto-telluric theory are based on the assumption that the electromagnetic variations are composed of a spectrum of harmonic terms. In order to compare the magnetic variations of a specific frequency, therefore, it is first necessary to separate the harmonic components of the record. An harmonic analysis, or Fourier analysis, enables a cyclic function to be expressed as a sum of sine terms with respective phase angles. Then the amplitude ratios of corresponding harmonics of the telluric and magnetic records might be used to give the apparent resistivity in accordance with equation (12). In general, it is not possible to separate the corresponding component harmonics by a simple inspection of the record, since two series composed of harmonic terms out of phase with each other will not have readily identifiable similarities.

On the records obtained, it was noticed that there were some portions which were cyclic. That is, similar features were repeated on the record at regular intervals. These particular portions were

analyzed according to standard methods described by Conrad and Pollack,[16] to give the harmonic components.

The cycles on the record chosen to be analyzed are divided into equally spaced time intervals (usually minutes or half-minutes.) The value of the ordinate at each interval is read from the records. These values are taken over as long an interval of the records as is possible, providing that the record is cyclic over the entire interval. The more cycles over which the data is read, the less is the effect of random fluctuations.

The data read from the records is entered on a "folded" form. "Folding" the data will reduce it to one average cycle.[17] Suppose that observations have been made for n cycles. Each cycle will have the same number, r, of individual observations so that altogether there will be $N = r \times n$ values of the ordinates. We arrange the N data,

$$y_o, y_1, y_2, \dots\dots\dots \quad y_{N-1}$$

in a table with r columns. Then the original data

---

[16] Conrad and Pollack, <u>Methods in Climatology</u>, p.119, Cambridge, 1950.

[17] <u>Ibid</u>., pp. 355-58.

y_i appear folded into n rows of r values each. In the last row, the averages, Y_i, for each column are entered. This last row of averages is the row of the ordinates of an average cycle of the disturbance chosen. The average row preserves the basic period and all its submultiples and weakens the random, low-persistance fluctuations.

If the average cycle thus obtained were periodic, the ordinates at each end of the cycle would be the same. Usually, though, this is not the case, because of the effect of non-cyclic or long term changes. Before the data is suitable for an harmonic analysis, this long term change must be removed from the cycle. This is done by applying a small linear correction to each value of the ordinates so that the two end points are the same. The center ordinate is used as the mid-point of the correction, and is left unchanged.

The non-cyclic correction makes the assumption that the drift is linear during the interval of the cycles analyzed. This would, in general, not be true. However, the non-linear error is small if several cycles are analyzed. The linear correction serves at least to eliminate the major portion of the change which would otherwise produce erroneous

results in the harmonic analysis.

The method of harmonic analysis employed uses twelve ordinates and gives five harmonics with their respective phase angles. The analysis is most conveniently carried out on a prepared form, such as the one adapted from Whittaker and Robinson and shown on Page 54.[18]

The twelve equally-spaced ordinates are taken from the average cycle. In the cases where the number of ordinates for which the cycle is computed is not a multiple of twelve, the average cycle was plotted and twelve ordinates were read off its graph.

Each pair of corresponding magnetic and telluric records were treated in a parallel manner. That is, the data was read from both records for the same periods, and the same ordinate spacing was used. The results of the analysis give the amplitude and phase angles of the first five harmonics of both fields. The amplitude ratios, (i.e., $\frac{E_x}{H_y}$) for each of the harmonics is then readily determined.

---

[18] Whittaker and Robinson, The Calculus of Observations, pp. 267-71, London, 1944.

HARMONIC ANALYSIS

d: ____________________ Station: ____________

	y_0 to y_6	____ ____ ____ ____ ____ ____ ____
	y_{11} to y_7	____ ____ ____ ____ ____
Sums	v_0 to v_6	____ ____ ____ ____ ____ ____ ____
Diffs	w_1 to w_5	____ ____ ____ ____ ____

	v_0 to v_3	____ ____ ____ ____	w_1 to w_3	____ ____ ____
	v_6 to v_4	____ ____ ____	w_5 to w_4	____ ____ ____
Sums	p_0 to p_3	____ ____ ____ ____	r_1 to r_3	____ ____ ____
Diffs	q_0 to q_2	____ ____ ____	s_1 to s_2	____ ____

	$p_1=$	$p_2=$	$q_2=$	$r_1=$		$q_1=$	$r_2=$	$s_1=$	s_2
.5x above	$h_1=$	$h_2=$	$l_2=$	$m_1=$	.866x above	$l_1=$	$m_2=$	$n_1=$	$n_2=$

	$p_0=$ $p_1=$	$q_0=$ $l_1=$	$p_0=$ $h_1=$	$p_0=$ $p_3=$	$m_1=$ $m_2=$
	$p_2=$ $p_3=$	$l_2=$	$p_3=$ $h_2=$	$h_1=$ $h_2=$	$r_3=$
col.					
col.					
Sum	$=12a_0$	$=6a_1$			$=6b_1$
Diff	$=12a_6$	$=6a_5$	$=6a_4$	$=6a_2$	$=6b_5$

$q_0 \mp q_2 =$ ____	$n_1 + n_2 =$ ____	$n_1 - n_2 =$ ____	$r_1 - r_3 =$ ____	
$= 6a_3$	$= 6b_2$	$= 6b_4$	$= 6b_3$	

Result: y=

Using equation(12), the amplitude ratio will give a value of apparent resistivity for the location. This value of apparent resistivity will vary with the period of the harmonic used if the subsurface is not homogeneous.

As was mentioned, equations (18), (19), and (20) may be used to construct theoretical curves, such as given by Cagniard.[19] Two sets of curves may be plotted from the data: 1) the apparent resistivity as a function of the resistivity of the upper and lower layers and the thickness of the upper layer (Figure 11), and 2) the phase difference between the electric and magnetic fields as a function of resistivity of the layers and the depth of the upper layer (Figure 12).

The theoretical curves plotted in Figures 11 and 12 are computed for an upper layer with resistivity of ten ohm-meters, and a depth, h, of one kilometer. The set of curves is computed for various values of the resistivity of the second layer, ρ_2. To apply these curves to a particular case where the resistivity and thickness of the upper

---

[19] Cagniard, Geophysics, 18:605-35 (1953).

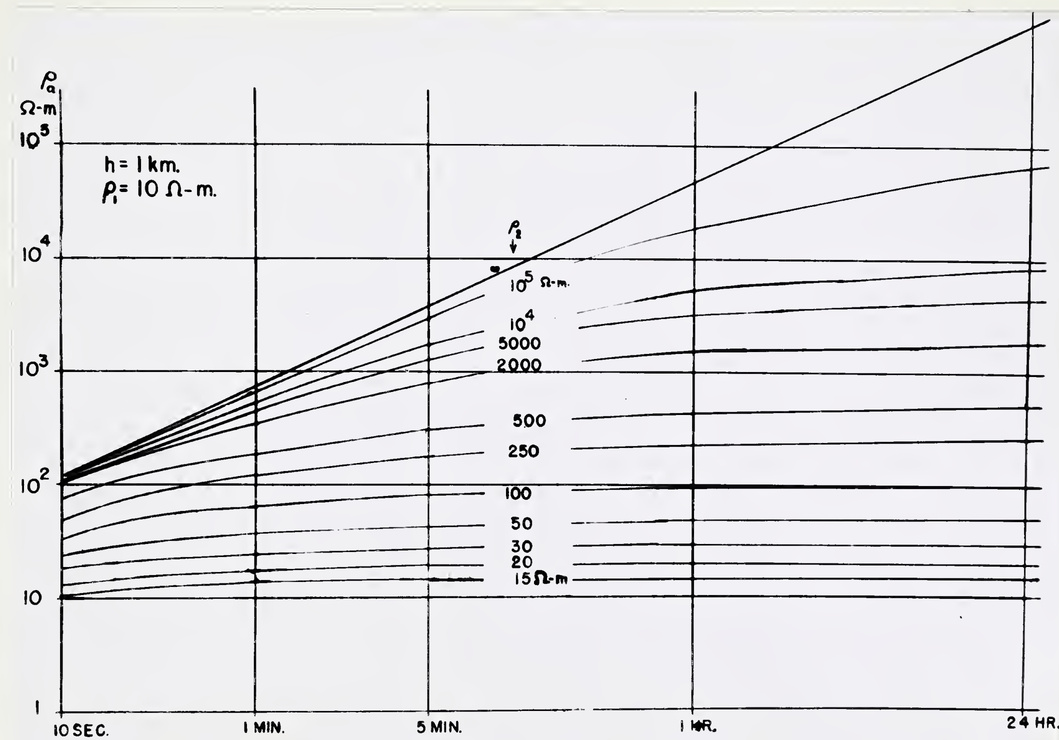

FIGURE 11

APPARENT RESISTIVITY FOR TWO-LAYERED SUBSURFACE

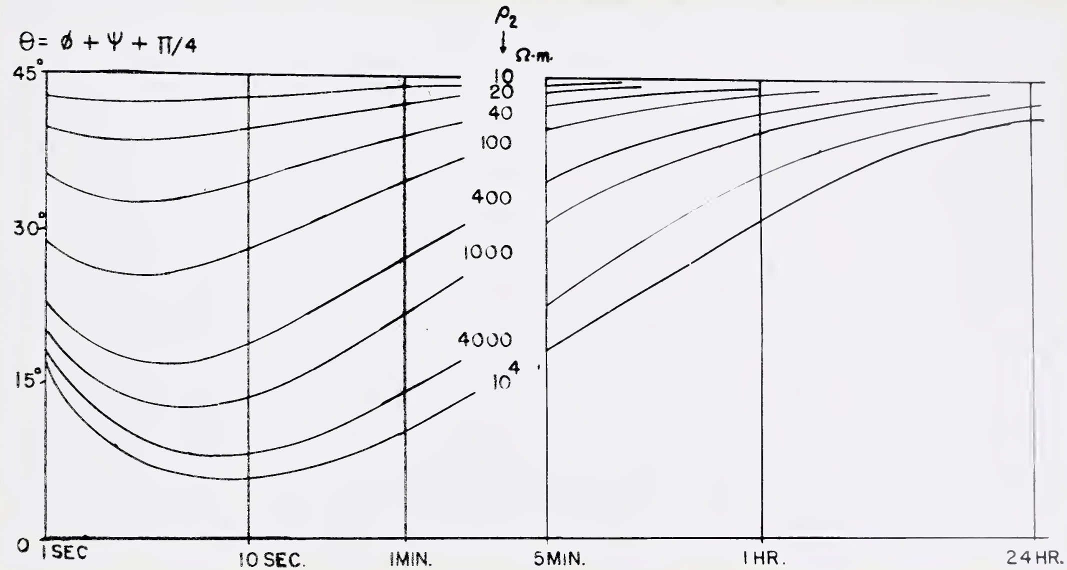

FIGURE 12

PHASE DIFFERENCE BETWEEN COMPONENTS FOR TWO-LAYERED SUBSURFACE

layer will not in general be the same as for the theoretical curves, the ordinate of the theoretical apparent resistivity curves (Figure 11) must be multiplied by $\frac{h^2}{\rho_1{}^2}$. Since the scales are logarithmic, both these factors may be applied to the theoretical curves by a shift of the axes.

To use the computed values of the apparent resistivity or phase difference, the values are plotted on transparent paper against period to the same scale as the theoretical curves. The calculated values on the transparent paper are then laid over the theoretical curves. The shift of the origin necessary to match the curves determines the value of the resistivity of the second layer and the depth of the upper layer.

The depth of the upper layer, h, may be read from the intercept of the curve of the observed data and the time axis. This time intercept, T_c, has a value:

$$T_c = \frac{64\, h^2}{10\, \rho_1} \qquad (23)$$

If a value of ρ_1, the resistivity of the upper layer is aasumed, the depth of the upper layer may be determined from equation (23).

CHAPTER FIVE

ANALYSIS AND INTERPRETATION

a) Analysis of a Typical Telluric Record

In this section, a typical telluric record will be treated to show the method of analysis in detail. The record chosen was that obtained at Clairmont, Alberta. Eleven usable time intervals were picked to give the values of the difference vectors from the field record. The time intervals are only "usable" if the records at both the base and field stations are clear and distinct. Due to calibration marks, off-scale surges of the field, and other disturbances, there were many portions of the records which could not be used for an analysis.

The difference quantities corresponding to the field changes for the chosen intervals are shown in Table II. The ordinates read off the records were first multiplied by suitable factors to give the results in units of millivolts per kilometer, which are the units usually used in telluric measurements. The amplitude of the base station difference vector was then computed and the difference quantities were all divided by this factor to

normalize them. After normalization, the data was suitable for presentation in graphical form. The graph of the base station unit circle and the field station ellipse are shown in Figure 13.

The ratio of the ellipse area to the area of the unit circle was computed using equation (21). That is,

$$S' = \frac{\triangle X_1 \triangle Y_2 - \triangle X_2 \triangle Y_1}{\triangle x_1 \triangle y_2 - \triangle x_2 \triangle y_1}$$

The data obtained from the field and base records, and the results of the computation of the field vector ellipse areas are shown on Table I.

The ellipse area for Clairmont, Alberta was found to be S' = 0.44, with a standard deviation of 0.26.

In addition to the ellipse areas, the value of the relative magnitude of the difference vectors was calculated for each of the intervals. The amplitudes of the two difference vectors were computed as:

$$|v_i| = \triangle z_i = \sqrt{\triangle x_i^{\,2} + \triangle y_i^{\,2}}$$

$$|V_i| = \triangle Z_i = \sqrt{\triangle X_i^{\,2} + \triangle Y_i^{\,2}}$$

Then the ratio of the two vectors,

$$\delta = \frac{\Delta z_i}{\Delta Z_i} \text{ .}$$

The amplitude ratios for Clairmont had an average value of 2.57 and a standard deviation of 0.75.

TABLE II

ANALYSIS OF TELLURIC RECORDS, CLAIRMONT, ALBERTA

i	Difference Components Δx_i	Δy_i	ΔX_i	ΔY_i	Δz_i	Normalized Components x_i'	y_i'	X_i'	Y_i'	Results S'	δ
	millivolts per kilometer										
2	22.4	-37.3	-14.3	4.4	43.5	.513	-.858	-.328	.102	0.22	2.9
3	-18.2	38.5	15.9	-7.7	42.6	-.427	.903	.373	-.181	0.95	2.4
4	13.4	-30.4	-10.8	7.4	33.2	.403	-.915	-.306	.222	0.83	2.5
5	22.0	-37.4	-2.6	-11.0	43.4	.507	-.862	-.060	-.330	0.46	3.8
6	12.5	-35.7	-11.2	8.9	37.8	.331	-.945	-.296	.236	0.20	2.6
7	-12.7	33.0	10.3	-8.9	35.4	-.359	.932	.291	-.251	0.41	2.6
8	11.4	-30.9	-11.5	9.2	32.9	.347	-.940	-.350	.280	0.32	2.2
9	-8.0	32.1	6.1	-8.2	33.4	-.242	.970	.184	-.148	0.58	3.2
10	10.8	-13.4	6.2	-9.5	17.2	.627	-.779	.360	-.552	0.09	1.5
11	-3.8	24.1	-5.2	11.0	24.4	-.156	.986	-.213	.450	0.34	2.0
12	-30.7	77.	17.4	-7.9	82.9	-.370	.930	.210	-.095	....	4.3

$S'_{av.} = 0.44 \pm 0.26$ $\delta_{av.} = 2.57 \pm 0.75$

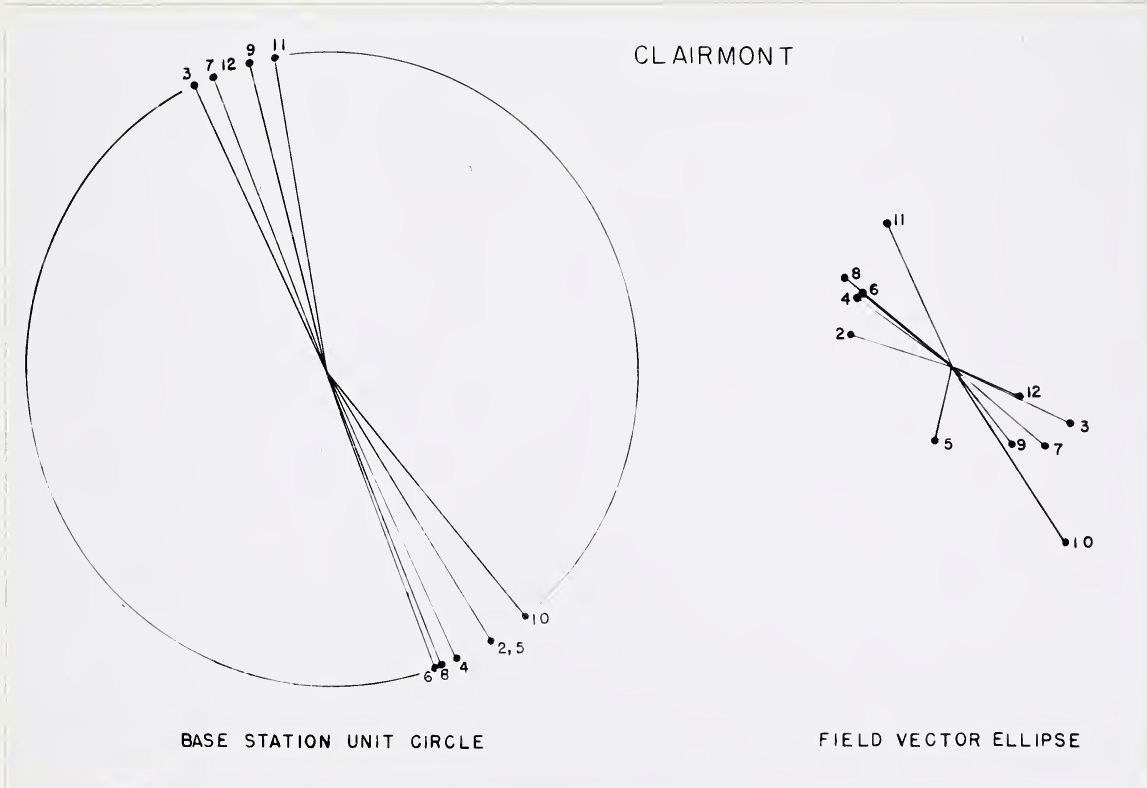

FIGURE 13

FIELD STATION ELLIPSE, CLAIRMONT, ALBERTA

b) Interpretation of the Telluric Analysis

Table III shows the results of the telluric analysis for each station computed in the manner presented in part (a) of this chapter. These results are shown graphically in Figures 14 and 15. Figure 14 shows the variation of the ellipse areas along the profile and the corresponding variations in the depth of basement. Figure 15 shows the variation in the relative amplitude of the telluric field along the profile.

The correlation coefficients between the basement depth and the values of the ellipse areas and relative amplitudes respectively were calculated according to methods described by Conrad and Pollack.[20] A correlation coefficient of zero signifies no correlation between two series, and a coefficient of one means that the series are identical or differ by a constant factor. A coefficient of 0.5 or less, when computed from only a few values, has little significance. Conrad and Pollack suggest that the correlation coefficient should be at least six times

[20] Conrad and Pollack, Methods in Climatology, pp. 245-247, Cambridge, 1950

TABLE III

STATION	DEPTH OF BASEMENT (feet)	ELLIPSE AREA	STANDARD DEVIATION	RELATIVE TELLURIC AMPLITUDES	STANDARD DEVIATION
Clairmont	11,500	0.44	0.26	2.57	0.75
Valleyview	10,200	0.82	0.58	2.58	1.07
Meanook	6,500	1.57	0.76	0.99	0.2
Vegreville	7,000	0.53	1.0	1.26	0.23
Vermilion	6,200	0.57	0.5	1.38	0.08
Leross	6,000	0.44	0.4	1.67	0.5
Yorkton	5,000	0.34	0.16	1.78	0.75
Franklin	3,500	1.82	1.9	0.94	0.33
Portage La Prairie	2,000	5.8	5.2	0.40	0.19

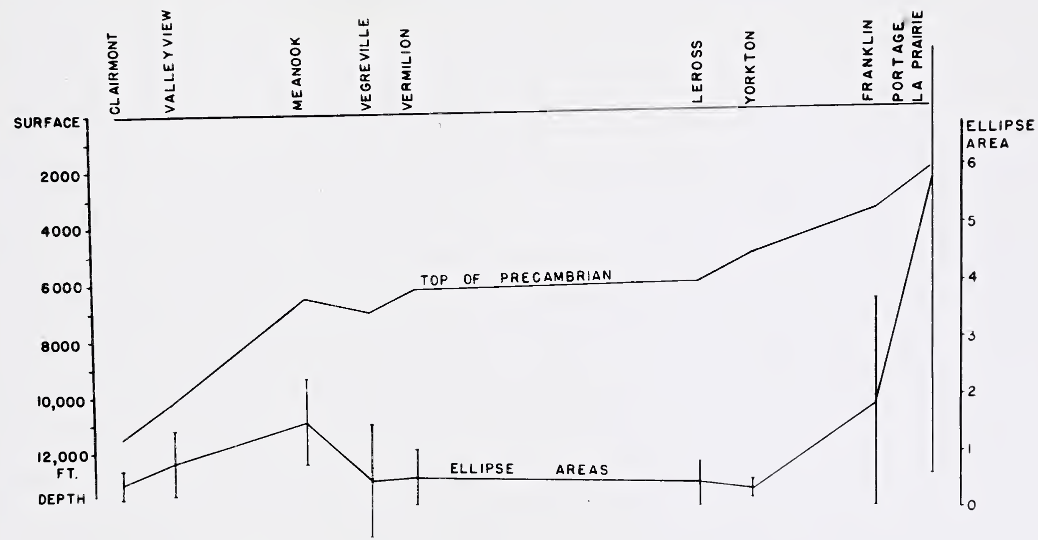

FIGURE 14

ELLIPSE AREAS ACROSS PRECAMBRIAN PROFILE

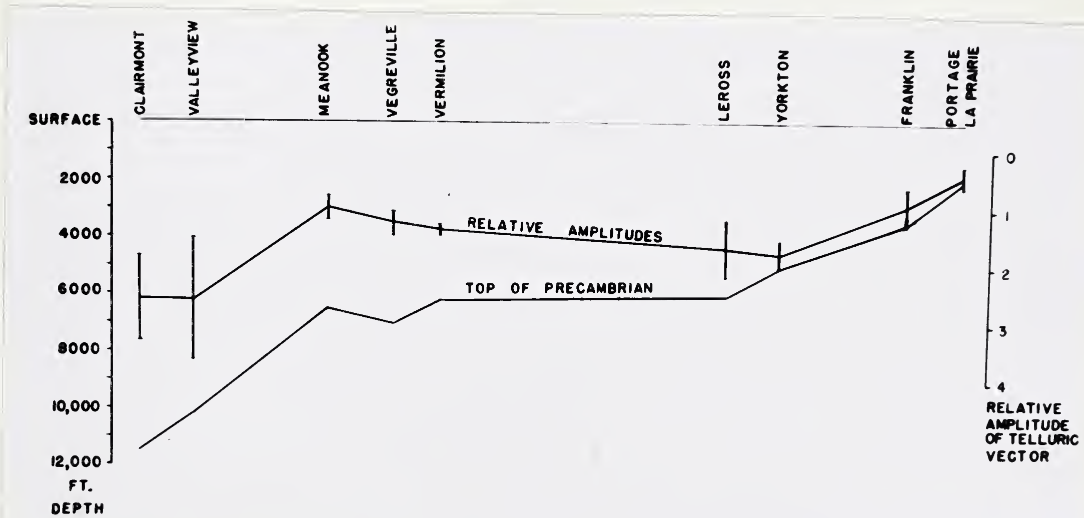

FIGURE 15

RELATIVE TELLURIC AMPLITUDES ACROSS PRECAMBRIAN PROFILE

its probable error to be considered significant.

The correlation coefficient between the ellipse areas and the basement was found to be 0.52 $\pm$ 0.12. The coefficient is 4.4 times the probable error. This value has little significance according to the criteria mentioned, so that the results of the ellipse area analysis across the profile would not seem to be useful as an indication of basement depth.

The correlation coefficient between the relative amplitudes and the depth of basement was found to be 0.68 $\pm$ 0.16. As before, the coefficient is 4.4 times its probable error. Because of the somewhat larger coefficient, however, there seems to be some correlation between the relative amplitude of the telluric field and the depth of the Precambrian formation.

Previous local surveys using the ellipse areas have shown a significant relation between the ellipse areas and subsurface structure. There are several probable reasons why the present investigation has not yielded more positive results with this method.

The actual records, themselves, for instance, were not well suited to the method of analysis. The sensitivity of the recorders used in the telluric

exploration was fifty millivolts full scale. This was not sensitive enough to be able to use the average variations in the telluric field for an analysis, so that only those variations which were somewhat larger than average could be used. At most stations, consequently, only a few intervals were analyzed with a consequent loss in the significance of the results. This problem was later eliminated by increasing the sensitivity of the recorders to ten millivolts full scale.

Another source of error in the telluric analysis was the timing of the records. While the field station records were timed about every half hour during the four hour recording period, the base station records were only timed about once every day. Since the paper drive at both stations fluctuated with line voltage, this introduced a timing error which could be quite substantial at some points on the records.

It can be seen from Figure 13 that the variations in the telluric field during the interval of the recording at Clairmont all lie along a single narrow band. It was found that during a four hour recording period, the direction of the telluric

vector did not vary enough to give a good graphical indication of the field vector ellipse. This difficulty was evident at the other field stations as well, so that it seems necessary to record over a longer period, say twenty-four hours, in order to determine the ellipse graphically.

The numerical computation of the field vector ellipse has disadvantages as well. The quotient in equation (21) is the difference between two quantities. If these quantities are somewhat similar, the quotient will be very small, and the probable error extremely high. In many cases, the quotient was nearly zero, which yielded an ellipse area of doubtful significance.

The relative amplitudes along the profile seem to have a better correlation with basement than the ellipse areas. It seems more practical to use relative amplitudes as a method of analysis than ellipse areas. The ratio of the amplitudes if far easier to calculate than the ellipse areas, since the ratio of the amplitudes may be calculated readily from the scale reading on the records. While the amplitude ratios cannot give an indication of the anisotropic nature of the soil, they are superior to the ellipse areas in that the possible

error is far less, and so the values obtained are probably more indicative of subsurface resistivity.

Similarities in the telluric records obtained at the most distant stations along the profile indicate the uniform nature of the telluric currents over a fairly large area on the surface of the earth. However, as the distance between the base station and the field station, increased, there was less and less similarity between the two sets of records. Thus, it seems probable that there is a practical limit to the area that can be investigated with this type of survey.

The poor correlation between the results of the two analyses and the depth of basement could possibly be due to a combination of two factors. At the distant stations, that is those at the eastern end of the profile, extraneous fluctuations in the telluric field might produce a record poorly suited to analysis. At the western end of the profile, where the basement was considerably deeper, the mean period of the field changes used in the analysis was not great enough to provide electromagnetic penetration to the depth of the top of the Precambrian formation. Perhaps if an analysis were undertaken

using field variations of a period long enough to penetrate to a suitable depth, more significant results could be obtained.

c) Analysis of a Typical Magneto-telluric Record

The magneto-telluric record chosen to be analyzed in detail was obtained at Meadow Lake, Saskatchewan. The original records are shown in Figures 16 and 17,

The values of the ordinates at half minute intervals were taken from the records for the interval from 0050 to 0101 GMT, June 5, 1957. This gave twenty-two values which were folded on a table on the assumption that there were two cycles of eleven ordinates. One average cycle was obtained from the folded data, and small linear corrections were applied to the average ordinates of the telluric and magnetic records respectively. This small correction was the difference between the end values of the average cycle. The final corrected average cycles of the telluric and magnetic components are shown in Figure 18.

Twelve equally spaced ordinates were taken from the average cycle of both records. The values of these ordinates were entered on an harmonic analysis form. The results of the harmonic analysis were:

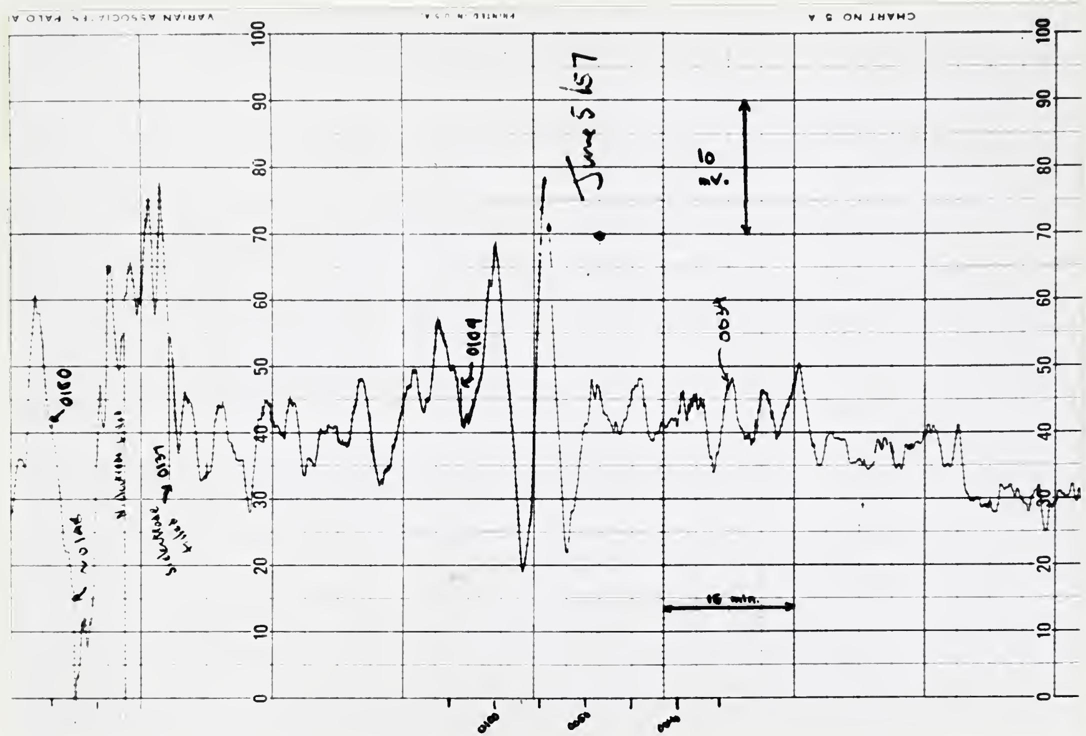

FIGURE 16

SELECTED TELLURIC RECORD, MEADOW LAKE

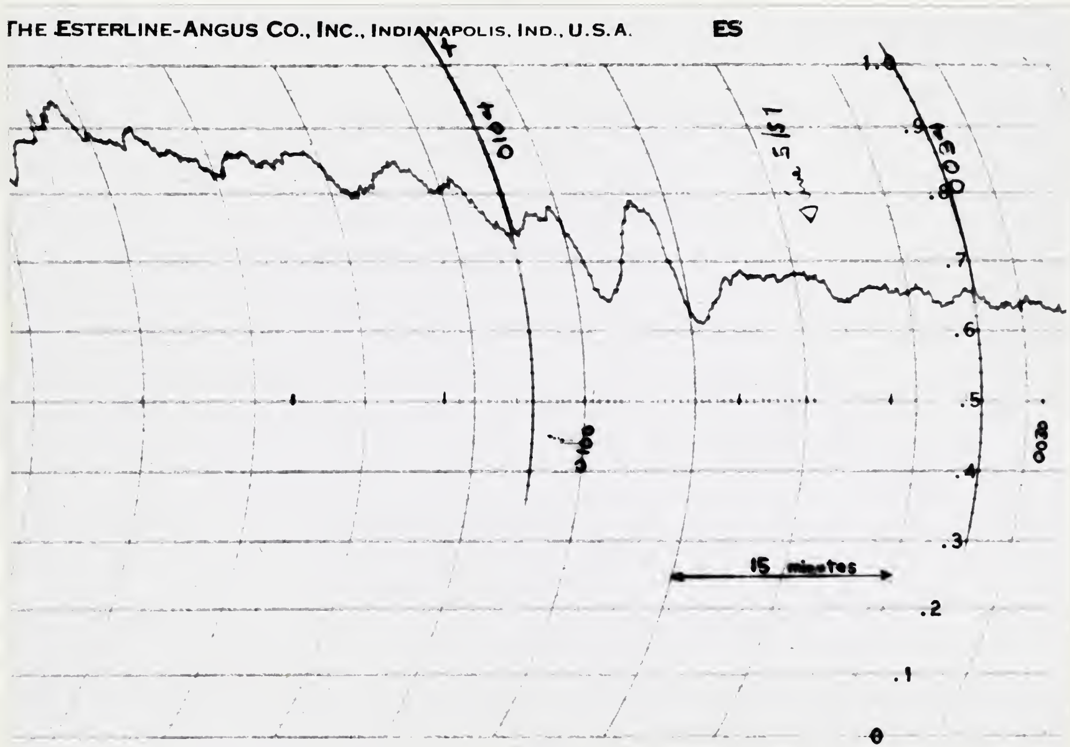

FIGURE 17

SELECTED MAGNETIC RECORD, MEADOW LAKE

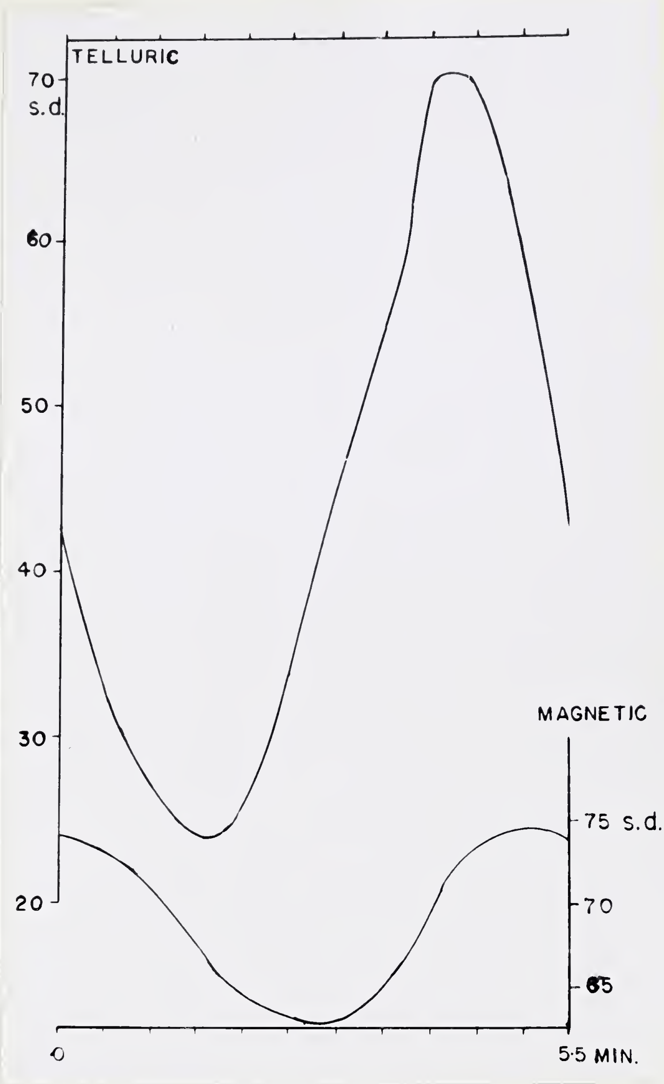

FIGURE 18

AVERAGE TELLURIC AND MAGNETIC CYCLES,
0050-0101, JUNE 5/57

$$E_x = 43.8 + 22.4 \sin(x + 171^\circ) + 3.5 \sin(2x + 237^\circ) + 1.6 \sin(3x - 78^\circ) + 0.3 \sin(4x + 38^\circ) + 0.8 \sin(5x - 12^\circ)$$

$$H_y = 69.8 + 6.0 \sin(x + 100^\circ) + 0.6 \sin(2x + 221^\circ) + 0.5 \sin(3x - 51^\circ) + 0.1 \sin(4x + 18^\circ)$$

Each of the above series is expressed in scale divisions, so that the numbers used must be multiplied by the scale factors of the recording instrument to convert the units to millivolts per kilometer and gammas, respectively.

The apparent resistivity which is to be determined is the value of ρ in equation(12). Equation (12) may be put into a form more convenient for the calculation of the apparent resistivity:

$$\rho_a = 0.2\ T \left(\frac{E_x}{H_y}\right)^2$$

In this equation, T is in seconds, E_x is in millivolts per kilometer, and H_y is in gammas. The apparent resistivity will be given in ohm-meters, The apparent resistivity calculated from the first harmonic, which has a period of 5.5 minutes, or 330 seconds, will be:

$$\rho_a = 0.2 \times 330 \times \frac{22.4 \times 16.4}{6.0 \times 3.4} \text{ ohm-meters}$$

$$\rho_a = 2.1 \times 10^4 \text{ ohm-meters.}$$

(The numbers "16.4" and "3.4" in the above example are the scale constants of the recorders.)

The results of the computations with the other harmonics are:

Second Harmonic: $T = 165$ seconds

$\rho_a = 2.6 \times 10^4$ ohm-meters

Third Harmonic: $T = 110$ seconds

$\rho_a = 5.3 \times 10^3$ ohm-meters

Fourth Harmonic: $T = 82.5$ seconds

$\rho_a = 3.5 \times 10^3$ ohm-meters

The phase differences between the harmonics are easily calculated by taking the difference between the corresponding phase angles. Thus, the phase differences are:

First Harmonic: 71°

Second Harmonic: 16°

Third Harmonic: 27°

Fourth Harmonic: 20°

These values of apparent resistivity and the phase differences for each harmonic may now be plotted

against the period of the harmonic to give observed curves to be compared respectively, with the apparent resistivity curve, Figure 11, and the phase difference curve, Figure 12. In order to obtain enough points for a significant observed curve, several analyses of various intervals must be undertaken to give a large number of points on the curve.

d) Interpretation of the Magneto-telluric Analysis

The results of the apparent resistivity computations for Molanosa, Meadow Lake, and Vega, are shown in Figures 20, 21, and 22 respectively. It is evident that the computed points do not fall along a curve similar to the theoretical curves shown in Figure 11.

Although the computed values are not suitable for curve-matching as described in the Theory, there seems to be a general increase in the apparent resistivity with period as is predicted by the theory.

Even if the computed data did lie approximately on a curve similar to the theoretical curves, however, it would not be possible to determine the depth of the Precambrian basement by curve-matching, since the range of frequencies used in the analysis is very small. There are no values less than one minute, and only two values greater than one hour. Consequently, the range for which the computed curve is determined is only a portion of the usable range, and is not enough to provide a unique match with the theofetical curves.

The problem of finding a match with the

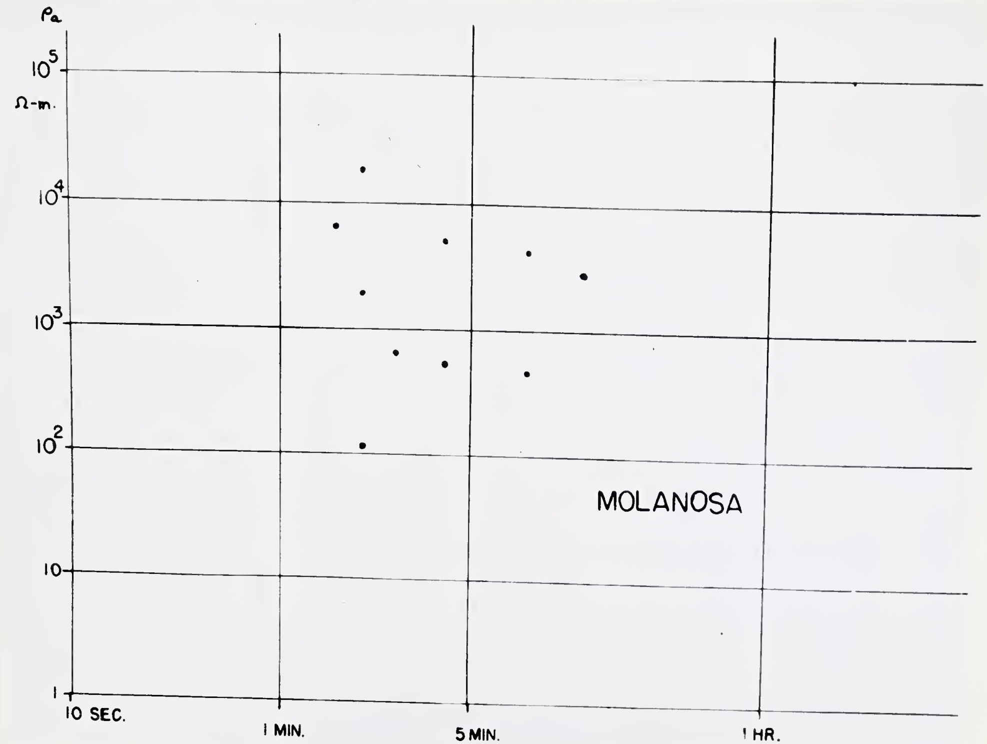

FIGURE 19

APPARENT RESISTIVITY, MOLANOSA

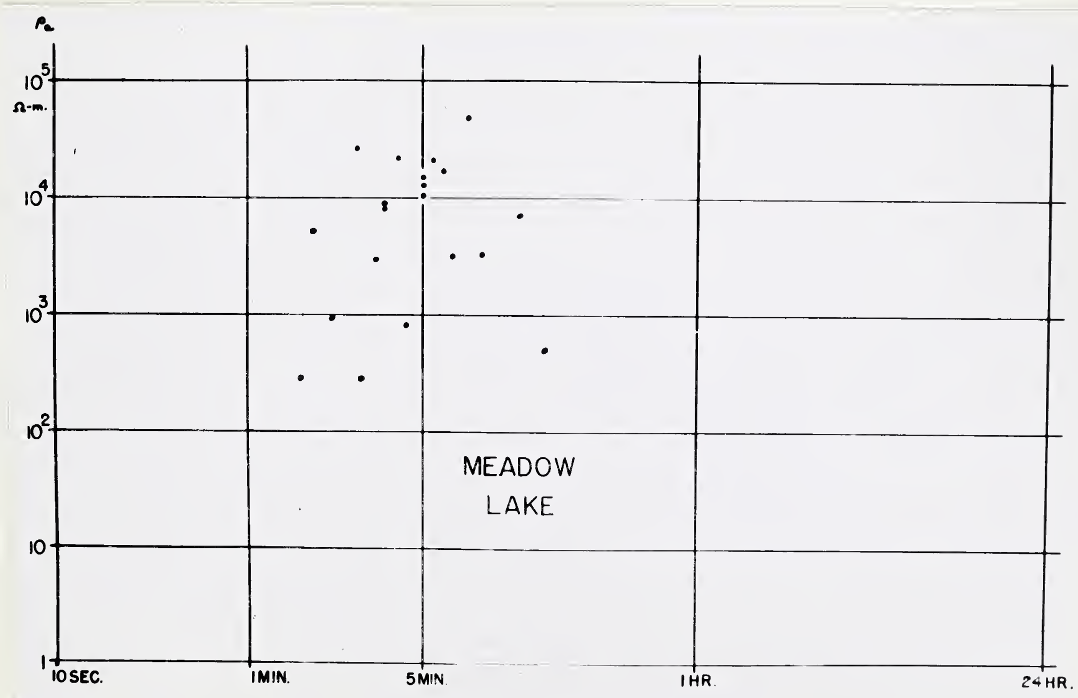

FIGURE 20

APPARENT RESISTIVITY, MEADOW LAKE

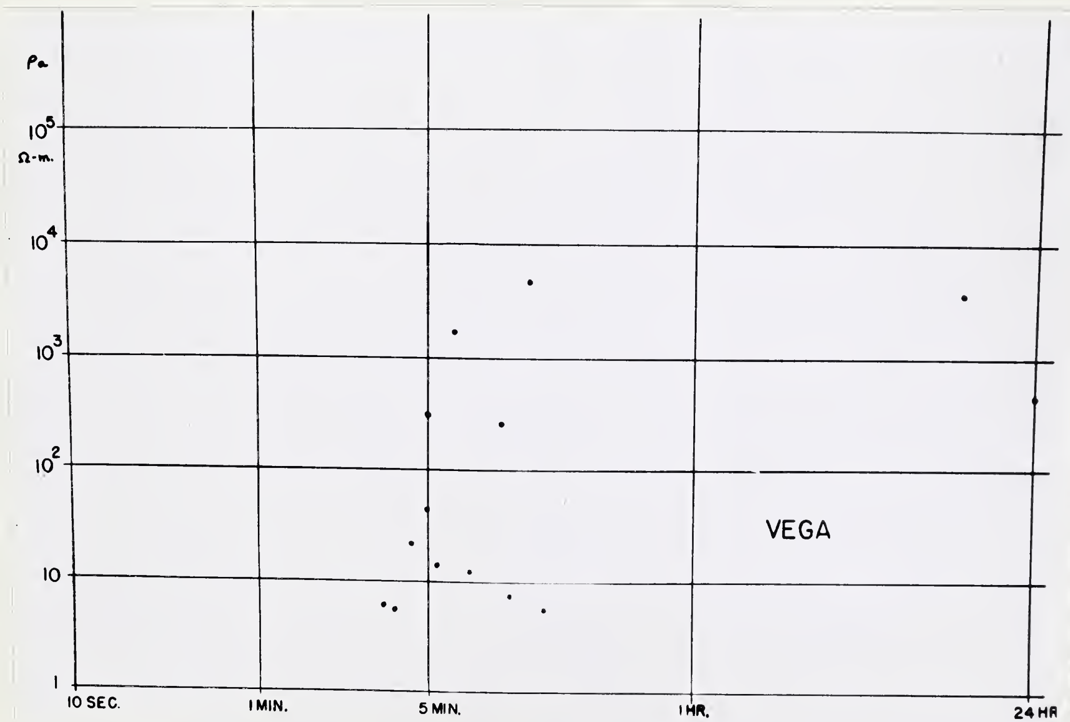

FIGURE 21

APPARENT RESISTIVITY, VEGA

theoretical curves is further exaggerated by the uncertainty of the resistivity of the upper layer. If the resistivity of the upper layer were known fairly accurately, then it would be possible to match even a small portion of the computed curve with the theoretical curve. If the resistivity of the upper layer is not known, however, there is a whole family of possible theoretical curves to which the computed portion may be matched.

The data as presented is not suitable for determining the subsurface properties as predicted by Cagniard. The limiting feature is the method of analysis applied -- especially the harmonic analysis. It is not possible to obtain the harmonics over a wide range of periods or for a large number of cases using the attempted method of analysis.

The small number of points on each of the graphs of computed data is due to the difficulty in finding portions of the record suitable to be analyzed, as well as the tedious computations necessary for the completion of a single analysis.

Although phase differences were calculated for each set of data, the results were inconsistent. There was no indication that the phase differences

as computed could be used for curve-matching. The computation of phase difference of a short period harmonic depend on an extremely accurate timing of the records. It is not believed that the accuracy between the magnetic and telluric records was any better than plus or minus fifteen seconds. This magnitude of uncertainty makes most phase difference results doubtful. For this reason, the results of the phase difference computations are not shown plotted for curve-matching.

CHAPTER SIX

CONCLUSIONS

a) Summary of Findings

It was found that the telluric prospecting as applied over a large scale structural feature showed little indication of subsurface structure.

Two methods of analysis were used with the telluric data obtained. The first, the method of ellipse areas, is favored by most of the previous investigators of telluric prospecting. It was found that the method of ellipse areas has less utility in those instances where the reliability of the data is not high. In these instances, an analysis using the relative amplitude ratios provides a more significant correlation with subsurface structure because of the less complex computations which are necessary.

In accord with the theoretical conditions predicted by Cagniard,[21] the telluric field measured over a large area showed a high degree of uniformity. The condition of uniformity is a

[21] Cagniard, <u>Handbuch Der Physik</u>, p. 462, Berlin, 1956.

necessary assumption for the development of the magneto-telluric theory, and also is in agreement with the results obtained by Schlumberger and Kunetz.[22] There were noticeable fluctuations present at the distant stations which were not present at the base station, however. As these non-uniform fluctuations increase with distance from the base, it imposes a limit on the scale of prospecting using the telluric field.

As a general result, telluric prospecting is less adaptable to large scale investigations than to the more local surveys where it is possible to obtain far greater control of the records between the field station and the base station.

It was not possible to match the computed resistivity curves with the theoretical curves in accord with the theory of Cagniard. The major limitation was that the analysis of the magnetic and telluric records did not cover a wide enough range of frequencies. The computed data lay between rather narrow frequency limits so that not enough of

---

[22] Schlumberger and Kunetz, C. R. Acad. Sci. Paris, 223:551 (1946).

the computed curve was defined in order to make a unique match with the theoretical curves. For this reason, the magneto-telluric prospecting investigated is not practical unless a method of analysis can be devised which makes use of a wider range of frequencies.

The investigation of the phase-difference curve matching was inconclusive because the magneto-telluric records were not timed accurately enough to give reliable results.

There was no indication that the theory as proposed by Cagniard is invalid. The restrictions of the analysis did not permit enough of the method to be carried out in order to determine the limits of Cagniard's method of subsurface investigation.

b) Recommendations

It is believed that more positive results might be obtained in telluric prospecting if the timing of the corresponding records were made more accurate. Also, it seems that there is a limit to the scale of telluric prospecting which can be undertaken due to the limits of the uniformity of the field over large areas.

It should be made certain that the variations used in the analysis have a period great enough so that their electromagnetic penetration is deep enough to be affected by the sub-surface structure investigated.

Before the magneto-telluric method of prospecting can be made practical, a method of analysis must be devised which will analyze a wider range of frequencies. It is not believed that a Fourier analysis according to standard methods will yield a wide range of frequencies. Furthermore, standard methods of harmonic analysis are only applicable to phenomena which are periodic. The magnetic and telluric records obtained have few distinctly periodic

intervals, and consist mainly of fluctuating, low persistance variations.

The method of analysis which seems to have the most promise of determining the amplitudes of the component frequencies of the records is that of a harmonic analyzer or spectrum analyzer using electronic methods.

A possible method of magneto-telluric prospecting would record the corresponding magnetic and telluric records simultaneously on magnetic tape. These could be re-played at any desired speed in the laboratory. The frequency range which may be analyzed can extend over an extremely large range by choosing the speed at which the records are played back.

Unless the computed curves are obtained over a wider frequency range, or the resistivity of the upper layer is known, the depth of the lower layer cannot be determined. With a wide frequency range in the analysis, the resistivity of the upper layer may be determined by the high-frequency variations. The resistivity of the lower layer may be determined by the low-frequency variations, which will penetrate deeper. If such a wide range of frequencies is

not available, it may be difficult to investigate large structural features by the magneto-telluric method, since it is often difficult to ascribe a single resistivity which is typical of the upper sedimentary layers as a whole.

BIBLIOGRAPHY

Boissonnas, E. and G. Leonardon, "Geophysical Exploration by Telluric Currents, with special referance to a Survey of the Haynesville Salt Dome, Wood County, Texas." Geophysics, 8:387-403, July, 1948.

Cagniard, L., "Basic Theory of the Magneto-telluric Method of Geophysical Prospecting." Geophysics, 18: 605-635, April, 1953.

__________, "Ellectricité Tellurique," Handbuch Der Physik, vol. XLVII, p. 407, Berlin: Springer Verlag, 1956.

Chapman, S. and J. Bartels, Geomagnetism, Oxford: The Clarendon Press, 1940. two volumes.

Conrad, V., and L. W. Pollack, Methods in Climatology, Cambridge: Harvard University Press, 1950, 459 pp.

Dahlberg, R., "An Investigation of Natural Earth Currents", Geophysics, 10:494, October, 1945.

Meek, J. H., and F. S. Hector, "A Recording Magnetic Variometer", Canadian Journal of Physics, 33:364-68, July, 1955.

Porstendorfer, G., Tellurik, Grundlagen und Anwendungen, Berlin: Akademie Verlag, 1954, 104pp.

Rooney, W. J., "Earth Currents", Terrestial Magnetism and Electricity, J. A. Fleming, editor, New York: McGraw Hill Book Company, 1939.

Schlumberger, M., "The Application of Telluric Currents to Surface Prospecting", Trans. Am. Geoph. Union, 3:271-277, 1939.

__________, and G. Kunetz, "Variations rapides simultanées du champ tellurique en France et á Madagascar", C. R. Acad. Sci. Paris, 223:551-53, 1946.

Tuman, V., "The Telluric Method of Prospecting and its limitations under certain geologic conditions", Geophysics, 16:102-114, 1951.

Wait, J. R., "On the Relation between Telluric Currents and the Earth's Magnetic Field", Geophysics, 19:281-89, April, 1954.

Whittaker, E. T., and G. Robinson, The Calculus of Observations, London: Blackie and Son, 4th Edition, 1944.

Wyckoff, R. D., "The Gulf Airborne Magnetometer", Geophysics, 13:182-208, April, 1948